Integrated Approach of CFD and SIR Epidemiological Model for Infectious Transmission Analysis in Hospital

Hiroaki Asanuma and Kazuhide Ito*

IGSES, Kyushu University
6-1 Kasuga-koen, Kasuga
Fukuoka, Japan

**Corresponding author: ito@kyudai.jp*

ABSTRACT
The indoor environment can play a significant role in the transmission of and exposure to various contaminants. In the case of some emerging aerial infections, such as those caused by influenza virus and tuberculosis virus, the airborne route of transmission is considered to be important for evaluating the health risk associated with exposure to contaminants. In this paper, we first present an analytical procedure for coupling the computational fluid dynamics (CFD)-based prediction of unsteady contaminant concentration distribution with a basic epidemiological model (here, SIR model) and show how this procedure can be used to predict exposure risk of people in hospital space.
In this study, we focus on the coupled simulation of unsteady and non-uniform distribution of infectious particle concentration and infectious risk, which directly indicates the changes in the population densities of *Susceptible* (*S*) and *Infective* (*I*) in a hospital space. The numerical analysis involved changes in the type of contaminant and infection probability and was performed for a hospital waiting space with a complicated geometry. The results showed a non-uniform distribution of (*S*) and (*I*) in such a space. Moreover, these results indicated the dependence on unsteady and inhomogeneous contaminant distribution.

KEYWORDS
Epidemiological model, SIR model, CFD, Hospital, Infectious transmission

INTRODUCTION

A majority of people in the developed world spend much of their lives in indoor spaces and hence the quality of indoor environments has a strong impact on their health. In indoor environments, huge numbers of contaminants exist and influence human health by long-term and short-term exposures. Concerning the sustainability of healthy and safe indoor environments, the control of airborne transmission by infectious bio-aerosols is one of the most important factors, especially in designing indoor spaces that many people use. This is especially true in hospitals, as the 2003 worldwide outbreak of severe acute respiratory syndrome (SARS) and a super-spreading event in a hospital ward in Hong Kong provided new understanding of the significance of indoor environmental control, especially ventilation in infection control.
The objective of this study is to develop an integrated numerical approach of computational fluid dynamics (CFD) and the classic SIR-type epidemiological model proposed by Kermack and McKendrich to analyze unsteady infectious transmission in hospital. Toward this end, first, we introduce an analytical procedure of coupled analysis of CFD-based prediction of unsteady contaminant concentration distribution and the classic SIR model to predict time-dependent exposure risk of people in enclosed spaces. Second, we demonstrate a coupled

CFD-SIR model simulation targeting a university hospital and investigate its effectiveness as a predictive tool for indoor air quality management.

OUTLINE OF EPIDEMIOLOGICAL MODEL

Various epidemiological models have been proposed to reproduce infection diffusion/propagation theoretically and mathematically in an enclosed space, and the SIR model proposed by Kermack and McKendrick is one of the basic epidemiological models that represent propagation of transmission. The classic SIR model consists of three differential equations coupling the changes in the population density of susceptibles (S), the population density of infectors (I), and the population density of those who recover to an immune state (R). The basic equations of the SIR model are indicated in Table 1 (equations (1) – (3)). The original SIR model defined by Kermack and McKendrick consists of only a reaction term, and equations (1) – (3) in Table 1 constitute an extended model including the diffusion term. β is the contact rate between susceptibles and infectors [1/person/s], and γ is the recovery rate [1/s]. The basic reproductive ratio R_o is defined from β and γ as shown in equation (5), and has been used widely as a parameter that describes the average number of new cases that an infector produces in a particular population. R_o>1 denotes that the infection rate is larger than the recovery or removal rate, which may lead to an epidemic and the spread of infection. R_o<1 indicates that the infection rate is smaller than the recovery rate, which may lead to an endemic situation. Here, the equation $S+I+R=N$ ($=S_0$) is used.

Table 1 Governing Equations of Epidemiological Model and Scalar Transport

[1] SIR-type epidemiological model (enhancing expression that added a diffusion term)

$$\frac{dS}{dt} = -\beta SI + \nabla(D_S \nabla S) \quad (1) \qquad \frac{dI}{dt} = -\gamma I + \beta SI + \nabla(D_I \nabla I) \quad (2)$$

$$\frac{dR}{dt} = \gamma I + \nabla(D_R \nabla R) \quad (3) \qquad \beta = p \cdot a\overline{C} \cdot \overline{\Delta} \quad (4)$$

[2] Basic reproduction number

$$R_o = \frac{\beta}{\gamma} S_0 \quad (5)$$

[3] Scalar transport equation for contaminant (Eulerian approach)

$$\frac{\partial \overline{C}}{\partial t} + \overline{u_i} \frac{\partial \overline{C}}{\partial x_i} - V_g \frac{\partial \overline{C}}{\partial x_3} = \frac{\partial}{\partial x_i}\left(\left(D_p + \frac{\nu_t}{\sigma_c}\right)\frac{\partial \overline{C}}{\partial x_i}\right) + S_0 \quad (6)$$

[4] Stokes' law of aerosol settling velocity

$$V_g = 2.46\left[\left(\frac{\rho_p}{\rho} - 1\right) d_p g\right]^{1/2} \quad (7)$$

PROCEDURE OF INTEGRATION ANALYSIS OF UNSTEADY CFD AND SIR MODEL

It was possible to predict unsteady and non-uniform concentration distribution by solving the scalar (aerial infectious contaminant) transport equation shown in equation (6) in Table 1 on the basis of flow field analysis by CFD. The time change of contact rate β distributions through unsteady concentration simulation in enclosed spaces and then equations (1)-(3) were solved using this time-dependent β (equation (4)). In the case that the infectors (I) emit infectious contaminant, namely, new contaminant sources, the source term S_0 that denotes the formation of scalar (new infectious contaminant) in equation (6) must be considered. In this study, S_0 and feedback from SIR analysis to scalar transport equation were disregarded. The SIR model in Table 1 is an enhancing model that considers diffusion terms in (S), (I), and (R), and the movement of susceptibles or infectious could be reproduced approximately by giving

the diffusion coefficient. In particular, when the grid size of CFD is set to larger than human scale, the numerical results of diffusion and propagation of infectors become reasonable analyses that reproduce an actual phenomenon. In this case, SIR parameters indicate population density, that is, [population/m^2], and the correction that corresponds to the width of the grid scale $\bar{\Delta}$ was built into β to ensure the correspondence of the dimension. In this study, unsteady and non-uniform concentration distribution was analyzed in three dimensions by CFD and governing equations of the SIR model were analyzed in two dimensions at a height of 1.6 m (y) from floor level (breathing zone level).

OUTLINE OF NUMERICAL ANALYSIS

Waiting Space in University Hospital

In this study, a waiting lounge of a university hospital is analyzed. Figure 1 shows an outline of the hospital space and a photo of its exterior.

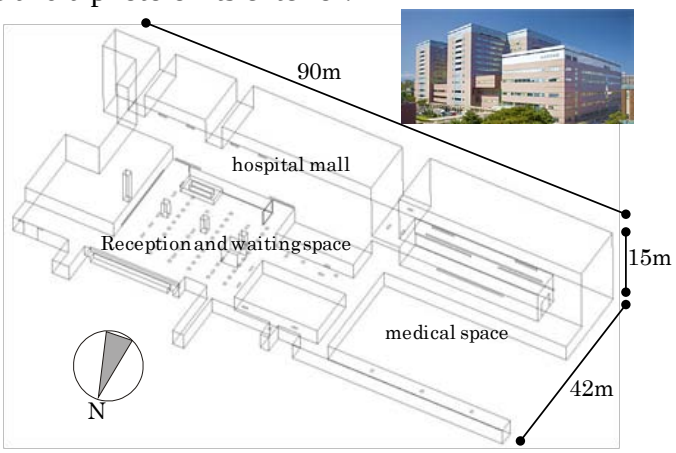


Figure 1 Perspective view of university hospital

The plan of this analytical space is 42 m × 90 m with a total floor area of 2020 m^2. The first floor of this hospital consists of three zones: (i) reception and waiting space on the north side, (ii) hospital mall on the south side, and (iii) medical space on the west side. The height of zones (i) and (iii) is 3.0 m and that of zone (ii) is 15 m. The heating, ventilation, and air-conditioning (HVAC) system was designed and constructed in accordance with these three zones. At (iii) medical space on the west side of the hospital, because the doors of each room were always closed and the HVAC system was also independent, zone (iii) medical space was excluded for the numerical analytical domain. Accordingly, the flow fields and contaminant distribution in (i) reception and waiting space on the north side and (ii) hospital mall on the south side were analyzed.

Multiple fan coil units (FCU) are arranged in zones (i) and (ii) in order to control indoor temperature. A total of 79 supply inlet openings of the air-conditioning system are installed on the ceiling and air is exhausted through lavatories in three places. The geometries of hospital space, furniture, and the supply inlet and exhaust outlet opening were simplistically modeled in order to avoid major effects on the prediction accuracy.

Numerical and Boundary Conditions and Cases Analyzed

Numerical and boundary conditions of CFD simulation and analyses of SIR model are shown in Table 2 and Table 3, which summarize the cases analyzed.

Flow fields were analyzed by the RNG k-ε model in the steady state condition. The SIMPLE algorithm was used with the QUICK scheme for the convective terms, and a second-order center difference scheme was used for the others. After steady flow field analysis under isothermal condition, unsteady aerosol contaminant concentration distributions were analyzed by solving ensemble-averaged scalar transport equation (6) in Table 1 based on a Eulerian

approach. Three types of contaminant were assumed: passive contaminant as gas phase, and aerosols of 10 μm and 50 μm in diameter. Concerning the aerosol dynamic equation, convection, diffusion, and gravitational settling were considered. Gradient zero of scalar was adopted as wall surface boundary condition. The contaminant was assumed to be an infectious bio-aerosol generated through 16 points of supple inlet opening of the air-conditioning system in zone (i) reception and waiting space constantly for two hours, and subsequently changed to zero concentration at the supply inlet positions. The contaminant concentrations of aerosol in this analysis were normalized by using supply inlet concentration of the air-conditioning system.

The total number of meshes was set to approximately 600,000 and unstructured mesh was used for the analysis. The analysis was carried out in three dimensions. The air inlet velocity from the air-conditioning system and the turbulent intensity were set to U_{in} = 0.5 m/s and 10%, respectively, in accordance with the drawing and specifications of the target hospital.

Table 2 Numerical and Boundary Conditions

Turbulent model	RNG k-ε model (3-dimensional Cal.)
Difference scheme	Convection Term: QUICK
Inflow boundary	U_{in} =0.5 [m/s] for n=4.36[h^{-1}], k_{in} =3/2× (U_{in}×0.1)^2, ε_{in} =$C_\mu^{3/4}$×$k_{in}^{3/2}$/l_{in}, l_{in} =0.03[m]
Outflow boundary	U_{out} = free slip, k_{out}= free slip, ε_{out} = free slip
Wall treatment	Velocity, generalized log low
Contaminant	Passive Contaminant Gravitational settling velocity: v_p= 3.0×10^{-3} m/s (D_p=10 μm), 7.5×10^{-2} m/s (D_p=50 μm) D_ϕ= 2.4× 10^{-12} [m^2/s] (D_p=10 μm), D_ϕ= 4.7× 10^{-13} [m^2/s] (D_p=50 μm),
SIR model	S_0=0.15[population/ m^2], I_0=5.0^{-4}[population/ m^2], p=1.67×10^{-4}[m^3/s], $\overline{A}$ = 0.25[m^2], μ=2.8×10^{-4}[1/s], D_S=D_I=2.8×10^{-4} [m^2/s]

Table 3 Cases Analyzed

Analysis case	Temp. condition	Aerosol Source	Aerosol diameter	Basic reproduction number R_0 [-]	Infectivity titer a [-]	Diffusion coeffi. D_S, D_I [m^2/s]
Case 1	Isothermal	HVAC system of zone (i)	Passive	15	670	0
Case 2				6	270	
Case 3				15	670	2.8×10^{-4}, 2.8×10^{-4}
Case 4				6	270	
Case 5			10 μm	15	670	0
Case 6				6	270	
Case 7			50 μm	15	670	
Case 8				6	270	

The transport equations of (S), (I), and (R) were analyzed in conjunction with solving the unsteady contaminant concentration distribution in hospital space. In the Wells-Riley model, which can evaluate airborne infection risk and predict the number of susceptibles (S) as a function of infectious contaminant concentration and exposure time by respiration, contaminant concentration is expressed as 'quanta' intensity. In modeling 'quanta' separately for simple scalar C transport and the infectivity titer a, independent analyses of scalar transport and infectious risk become possible, as shown in equation (4) in Table 1. In this analysis, infectivity titer "a" was set at two levels: 670 [-] and 270 [-], and these values agree with the quanta concentrations of the Wells-Riley model of 670 [quanta/m^3] and 270 [quanta/m^3], respectively.

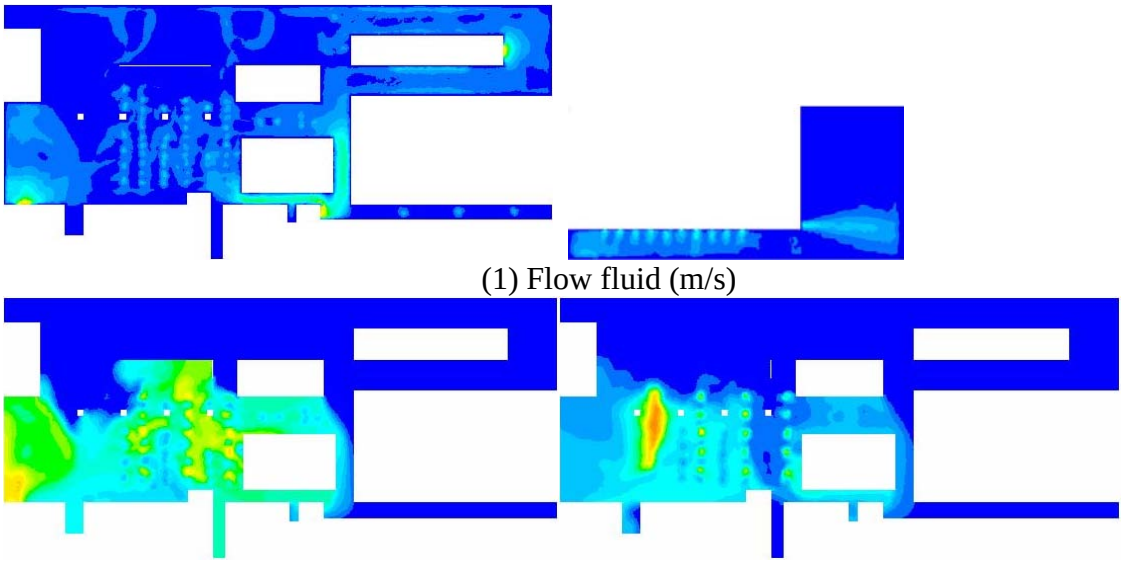
In this study, unsteady and non-uniform concentration distribution was analyzed in three dimensions by CFD and governing equations of the SIR model were analyzed in two dimensions at the height of the breathing zone (1.6 m height).
CFD simulation is preferable to understand a non-uniform distribution in a comparatively short time scale. On the other hand, SIR-type epidemiology model is often used for infection transmission prediction for a long time scale, such as weeks or months. In this analysis, the condition for highly infectious contaminant generated in the target space was assumed, and unsteady analyses were carried out for 6 hours in consideration of the time usually spent in the hospital lobby.
In this analysis, the population of infectors did not recover (γ=0) and they were assumed to die at a constant rate after infection. γ in equation (2) was replaced with death rate (μ) in this analysis. The initial population of susceptibles S_0 was set at 0.15 [population/m^2], which corresponds to a total of 300 people in hospital. I_0 was assumed to be 5×10^{-4} [population/m^2], which corresponds to 1 infected person in the hospital. The contact rate (β) was calculated by infectivity titer (a) and prediction results of contaminant concentration in the breathing zone (C at y=1.6 m). The basic reproductive ratio (R_o) was set at two levels: R_0=15, which corresponds to measles, and R_0=6, which corresponds to rubella, and are extremely high values.
The movement of susceptibles or infectors could be reproduced approximately by giving the diffusion coefficient. This diffusion coefficient was estimated by using representative length scale in hospital space, nominal time constant (time scale estimated by space volume and ventilation rate), and diffusion speed.

RESULTS

Here, the results of flow fluid in hospital waiting space, concentration distribution of infectious contaminant, and population density of susceptibles (S) and infectors (I) by coupled simulation of unsteady CFD and SIR-type epidemiological model are shown in series.

Flow field

Figure 2(1) shows the result of flow fluid analysis of x-z plane at y=1.6 m height and representative x-y section with isothermal condition. The flow field in hospital waiting space was almost stagnant and air velocity was below 0.1 [m/s] except for the vicinity of jet flow from a supply inlet (U_{in} =0.5 [m/s]). A non-uniform distribution of flow field was confirmed.



(1) Flow fluid (m/s)

(2) Contami. distribution in Case 1 (passive) (3) Contami. distribution in Case 7 (Dp=50 μm)

Figure 2 Results of flow fluid and contaminant distribution in hospital (y=1.6 m, t=2 hours)

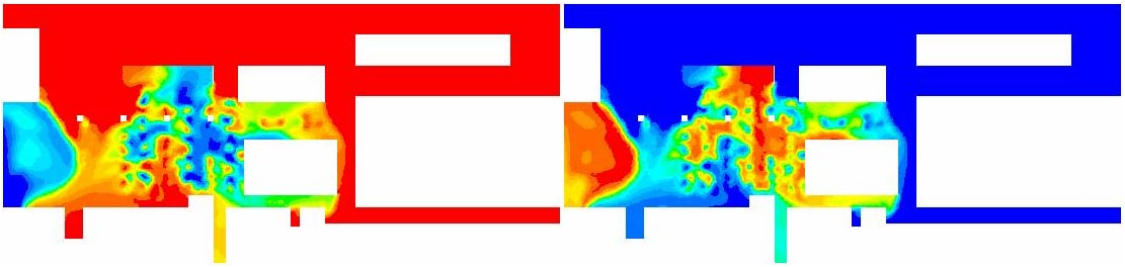
Contaminant concentration distribution

Figures 2(2) and (3) show the results of contaminant concentration distribution at two hours after the contaminant generation start in two representative cases: Case 1 for passive contaminant and Case 7 for aerosol particle (Dp=50 μm). The results of Case 5 (Dp=10 μm) were almost the same distribution as Case 1 and are omitted in this paper. Infectious contaminant was generated through the supply inlet opening of the air-conditioning system in zone (i). The concentrations in Figure 2(2) and (3) were normalized by supply inlet concentration (C_{in}).

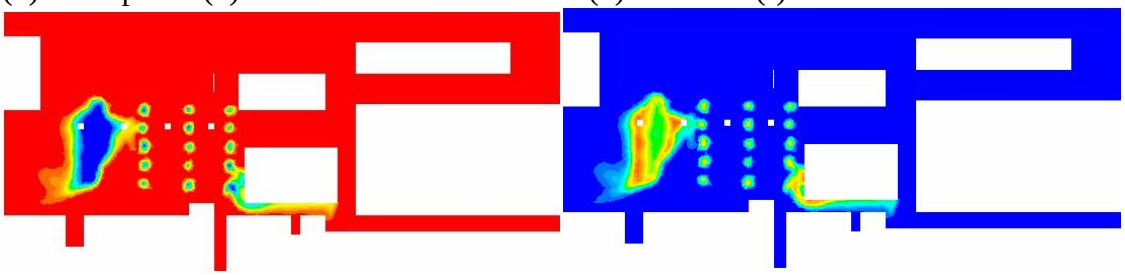
Non-uniform concentration distributions were formed in the space for both bases. In the case of passive contaminant (Case 1 in Figure 2(2)), the contaminant stayed and widely diffused within zone (1) of the hospital waiting space. On the other hand, in the case of Dp=50 μm size aerosol (Case 7 in Figure 2(3)), non-uniformity of concentration distribution became clear and A remaining in the high-concentration zone was confirmed in the center of the zone (i) compared with that in Case 1. This was because of the effect of gravitational sedimentation of Dp=50 μm size aerosol.

Transmission analysis and distributions of (S) and (I)

Figure 3 shows the population density of susceptible (S) and infectious (I) distributions in the x-z plane at y=1.6 m height at 2 hours from the start of analysis for two representative cases: Case 1 and Case 7.



(1) Susceptible (S) in Case 1 (2) Infective (I) in Case 1



(3) Susceptible (S) in Case 7 (4) Infective (I) in Case 7

Figure 3 Distributions of population density of Susceptible (S) and Infective (I) (y=1.6 m, t=2 hours)

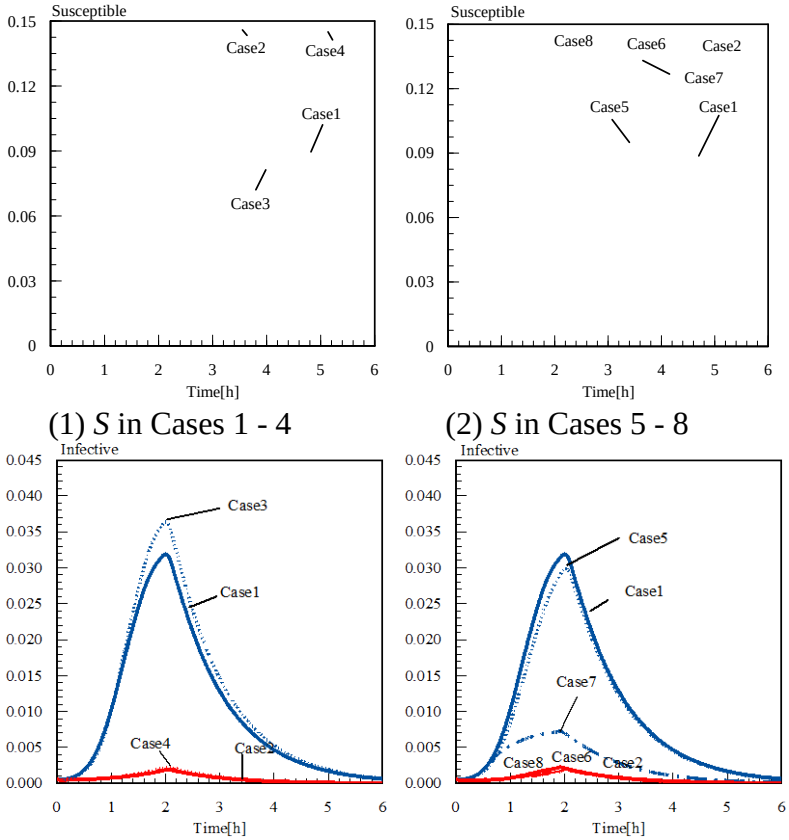
In Case 1 as shown in Figures 3(1) and (2), the distributions of (S) and (I) were similar, which was because (S) and (I) were assumed to be motionless and converted from (S) to (I) at the same points. (S) and (I) exhibited inhomogeneous distributions caused by non-uniform distribution of contaminant. In other words, there were inhomogeneous distributions of infection risk in this hospital waiting space. In Case 7, for which the dispersion of Dp=50 μm size aerosol was assumed, the distributions of (S) and (I) were not spread and were caused only in the high concentration region at the center of zone (i) corresponding to Figure 2(3).

Figure 4 shows the time series of average values of (S) and (I) for each case. For Case 2, Case 4, Case 6, and Case 8, for which the infectivity titer "a" was set at a higher value (R_0=6 and a=270), contact rate β became small in a short time scale and the time series of (S) and (I) showed constant profile. On the other hand, for Case 1, Case 3, Case 5, and Case 7, for which

the infectivity titer "a" was set at a higher value (R_0=15 and a=670), space-averaged (S) and (I) greatly changed and, especially the time profile of (I), had a clear local maximum value and then decreased gradually. Considering the diffusion of (S) and (I) in Case 3 and Case 4, the number of (S) was decreased and that of (I) was increased compared with those for Case 1 and Case 2. This is because the mixture of (S) and (I) was accelerated and the reaction/generation term (βSI) of the transport equation of (I) was increased.

Compared with Case 1 and Case 7, the time profiles of (S) and (I) were completely different in these two cases and the characteristics of unsteady and non-uniform concentration distribution of infectious contaminant were confirmed to impact on infectious risk in hospital space.

The peak value of (I) in Case 1 was 0.032 [population/m^2], which corresponded to 64 people infected.



(1) S in Cases 1 - 4 (2) S in Cases 5 - 8

(3) I in Cases 1-4 (4) I in Cases 5-8

Figure 4 Time series of population density of (S) and (I) for each case
(Vertical axis indicates space-averaged value of population density)

DISCUSSION

In this study, numerical analyses of unsteady and non-uniform concentration distribution by three-dimensional CFD and the procedures of coupled simulation between CFD and SIR-type epidemiological models were carried out. On the basis of these proposed procedures, the time dependence and distribution of infectious risk in indoor environments could be predicted quantitatively. Contaminant concentration in the breathing zone was strongly influenced by the setting of the ventilation condition and position of contaminant source, and therefore contact rate (β) was estimated as unsteady and inhomogeneous; then, space distributions of (S) and (I) were greatly changed in accordance with the boundary condition.

As shown in equations (1)-(3), constant (linear) and isotropic diffusion coefficients were adopted to reproduce movement of residents in hospital space. In order to reproduce human

behavior more accurately, it is necessary to adopt a non-isotropic diffusion model and analysis by cellular automata. This will also be investigated in future.

CONCLUSION

In this paper, an integrated analytical procedure of unsteady CFD simulation and SIR-type epidemiological model for infectious transmission in hospital space was proposed and the results of numerical simulation were also demonstrated.

Under the assumption of a constant linear rate of pulmonary ventilation rate and contaminant concentration in the breathing zone, contact rate β can be directly estimated. Through the analysis of infectious contaminant concentration level and non-uniform distribution in a large hospital space with CFD, the prediction of the number changes of (S), (I), and (R) becomes possible through the analysis of β.

As a result of this analysis, the contaminant concentration in the breathing zone was found to be strongly dependent on the characteristic of target contaminant, that is, the aerosol diameter, and then the distributions of (S) and (I) were greatly changed in accordance with unsteady contaminant conditions. We believe that the prediction procedures of coupled analysis of CFD and SIR model are effective for infectious risk assessment in indoor environments with reasonable accuracy.

ACKNOWLEDGEMENTS

This research was partly supported by a Grant-in-Aid for Scientific Research (JSPS KAKENHI for Young Scientists (S), 21676005). The authors would like to express special thanks to the funding source.

REFERENCES

[1] Li, Y., Huang, X., Yu, I. T. S., Wong, T. W., Qian, H., Role of air distribution in SARS transmission during the largest nosocomial outbreak in Hong Kong, ***Indoor Air***, 15 (2) pp83-95, 2004

[2] Kermack, WO. and McKendrick, AG., A contribution to the mathematical theory of epidemics, ***Proceedings of the Royal Society of London Series*** *A, 115*, pp 700-721, 1927

[3] Fine,P.E.M., Herd immunity: history ,theory, practice, ***Epidemiological Review*** *15(2)*: pp265-302, 1993

[4] Ito, K., Numerical prediction of Infection Process by Coupled CFD and SIR model in Indoor Space, *Indoor Air 2011*, The 12th International Conference on Indoor Air Quality and Climate, Austin, Texas, June 5-10, 2011, Manuscript number (a258-2)

[5] Fukuoka, T., Ito, K., Exposure Risk Assessment by Coupled Analysis of CFD and SIR model in Enclosed Space, *AIVC 2010*, Low energy and sustainable ventilation, Technologies for green buildings, Seoul, Korea, Manuscript number (1A-3)

[6] Yakhot, V. and Orszag, S.A., Renormalization group analysis of turbulence, I. Basic theory. ***J. Sci. Comput.***, ***1***: 3-51, 1986