GEOMETRIC OPTIMIZATION OF FENESTRATION

Jonathan Wright and Monjur Mourshed

Department of Civil and Building Engineering,
Loughborough University, Loughborough,
Leicestershire, LE11 3TU, UK

ABSTRACT

Fenestration and its design have a significant impact on the energy use associated with the artificial lighting, heating and cooling of a building. To date, research into window optimization has been for windows that are constrained to have a regular geometric shape and position. This paper describes an approach in which a building façade is divided into a number of cells, each cell having one of two possible states, a solid wall construction, or a window. A Genetic Algorithm search method has been used to optimize the state of each cell, with the number or aspect ratio of the windows being constrained were desirable. It can be concluded that the approach results in design solutions that have interesting and innovative architectural forms and which minimize building energy use.

INTRODUCTION

Fenestration and its design have a significant impact on the energy performance of buildings; it impacts on daylight penetration, artificial lighting energy use, and heating and cooling energy use. To date, optimization methods have been used to minimize building energy use by optimizing the dimensions of the windows and shading elements, but with the window shape always being rectangular and only minor movement in the position of the windows on the façade being allowed (among others; Caldas and Norford, 2002). Although this approach can result in reduced building energy use (and associated carbon emissions), the fact that the shape and positions of the windows on the façade are fixed limits the extent to which energy use can be reduced. Restricting the shape of the windows to be rectangular also inhibits the architectural form of the fenestration.

The objective of the research presented in this paper is to investigate an approach to fenestration optimization in which the shape, number, and position of windows can be optimized for minimum building energy use. The optimization is implemented by dividing the building façade into a number of small rectangular cells, each cell being defined by the optimization to be either a solid construction or glazed. Since the designer may wish to maintain some control over the general shape and number of windows, metrics have been developed for

the cellular window structure that allow the aspect ratio and number of windows to be constrained during the optimization.

The cellular optimization approach can result in design solutions that resonate with the form of fenestration developed by Le Corbusier in the design of the Chapel of Notre Dame du Haut (a widely known example of modern architecture). The South façade of this building, known as the 'wall of light', has windows of varying size that are randomly placed on the wall (Figure 1). It is considered by some (Buser, 2005), that this results in a fenestration design that goes beyond the general purpose of illuminating the interior space and enables an aesthetic interplay of light and form that embodies a feeling of intimacy and an aura of mystery.

Figure 1,
Façade Detail, Le Corbusier's Chapel of Notre Dame du Haut
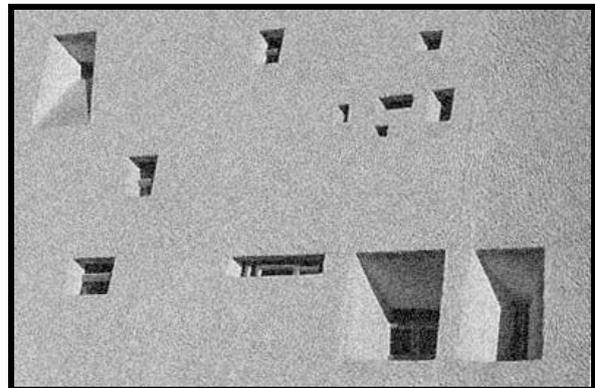
OPTIMIZATION APPROACH

The optimization of building fenestration investigated in this paper is based on the building façade being divided into a number of rectangular cells. Each cell has one of two possible states, a window element, or an opaque construction element. The number and location of the cells having a window construction is set by an optimization algorithm, the objective of the optimization being to minimize building energy use.

In order to allow the designer to have some control over the form of windows generated, several metrics

have also been developed that can be used both as design constraints during the optimization, and in the analysis of the design solutions.

Window Metrics

Five metrics have been defined for use as design constraints and/or in the analysis of solutions:

- the number of windows;
- window area;
- window aspect ratio;
- window density;
- the "centre of gravity" of the windows.

Number of windows: Figure 2 illustrates a façade of 15 cells wide by 8 cells high (a total of 120 cells). The individual window cells are shaded (blue), and have a bold outline. A "window" is defined by a set of adjoining window cells, there being 3 such "windows" in Figure 2. Note also that as indicated for "Window 1", cells that are adjacent at their corner point are considered to be part of the same "window".

Figure 2,
Cellular Window Façade
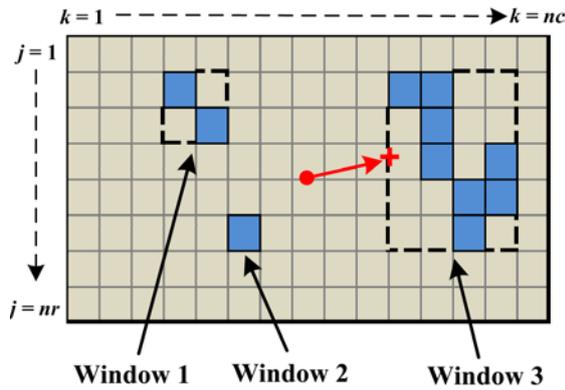
Window area: the total window area is expressed as a percentage of the total façade area and is defined by:

$$A_{windowTotal} = \frac{N_{total}^{windowCells}}{nt} \times 100 \qquad (1)$$

where; $A_{windowTotal}$ is the total percentage window area (%), $N_{total}^{windowCells}$ is total number of window cells, and nt is the total number of cells on the façade. The total percentage area of all windows in Figure 2 is 9.2%.

Window aspect ratio: aspect ratio has been defined as height divided by width. The height and width of a window is described by the rectangular box that forms a boundary on all cells of the "window". The bounding-box of the windows in Figure 2 is

illustrated as a dashed line. The aspect ratio of a window i, is given by:

$$AR_i = \frac{N_i^{cellsHigh}}{N_i^{cellsWide}} \qquad (2)$$

where; AR_i is the aspect ratio of window i, $N_i^{cellsHigh}$ is the number of cells forming the height of the bounding-box, and $N_i^{cellsWide}$ is the number of cells forming the width of the bounding-box. As such, "Window 1" in Figure 2 has an aspect ratio of 1.0, "Window 2", 1.0, and "Window 3", 1.25.

Window density: the majority of conventional windows are completely rectangular. The extent to which the optimized "windows" are rectangular is defined by the window density, this being a measure of the extent to which the window cells fill the rectangular bounding-box:

$$D_i = \frac{N_i^{windowCells}}{N_i^{totalCells}} \times 100 \qquad (3)$$

where; D_i is the percentage density (%), of "window" i, $N_i^{windowCells}$ is the number of window cells associated with "window" i, and $N_i^{totalCells}$ the total number of cells associated with "window" i. "Window 1" in Figure 2 has a density of 50%, "Window 2" 100%, and "Window 3" 40%.

Window position: the bias in position of the windows is measured through the mean "centre of gravity" of all windows. This is given as a vertical and horizontal coordinate position:

$$CoG_v = \frac{\sum_{j=1}^{nr} N_j^{windowCells} \times j}{N_{total}^{windowCells}} \qquad (4)$$

$$CoG_h = \frac{\sum_{k=1}^{nc} N_k^{windowCells} \times k}{N_{total}^{windowCells}} \qquad (5)$$

where; CoG_v and CoG_h are the vertical and horizontal coordinate positions respectively, j and k are the index of the cellular row and column respectively (Figure 2), $N_j^{windowCells}$ and $N_k^{windowCells}$ are the number of window cells in row j and column k respectively, and $N_{total}^{windowCells}$ is the total number of window cells on the façade.

The centre of gravity of the windows in Figure 2 is marked with a $+$; the associated arrow indicates the direction and distance of the centre of gravity from the centre of the façade.

Problem Formulation

The optimization problem is defined by:

$$\min f(X) = \sum_{l=1}^{nl} \left(Q_l^{heat} + Q_l^{cool} + Q_l^{elect} \right) \qquad (6)$$

subject to:

$$g_i(X) \leq 0.0; \quad \forall i \in \{1, \cdots, ng\} \qquad (7)$$

where; Q_l^{heat}, Q_l^{cool}, and Q_l^{elect} are the heating, cooling and electrical energy use (kWh) at building load condition l; nl is the number of load conditions; the electrical energy use including the energy used in artificial lighting.

$g_i(X)$ is constraint function i, there being a total of ng constraint functions. The constraint functions are formulated from the window metrics; in the experiments described in this paper, two metrics have been used to constrain the design solutions; the total number of windows; and the window aspect ratio.

X is a vector of discrete problem variables which is mapped to a nc by nr matrix of binary decision variables $x_{k,j}$, one variable for each cell on the façade:

$$X \rightarrow \begin{pmatrix} x_{1,1}, x_{2,1}, \cdots\cdots, x_{nc,1} \\ x_{1,2}, x_{2,2}, \cdots\cdots, x_{nc,2} \\ \vdots \\ x_{1,nr}, x_{2,nr}, \cdots\cdots, x_{nc,nr} \end{pmatrix}; \qquad (8)$$

$$x_{k,j} \in \{0,1\};$$
$$\forall k \in \{1, \cdots, nc\};$$
$$\forall j \in \{1, \cdots, nr\}$$

where, nc is the number of columns of cells, and nr the number of cell rows on the façade (Figure 2). A decision variable value of 0 results in cell (k, j) having a solid opaque construction element, whereas a value of 1 results in a window element.

Optimization Algorithm

A binary encoded Genetic Algorithm (GA) has been used as the optimization method in this research. Not only have GA been shown to be effective in solving building optimization problems (Wetter and Wright, 2004), but the GA binary encoding of the variables naturally lends itself to the characteristics of the cellular window optimization problem variables, these also being binary.

The form of GA implemented here is based on:
- random initialization of the problem variables;
- a Gray binary encoding of the variables (De Jong, 2006);
- binary tournament selection (De Jong, 2006), with the solution "fitness" being assigned by the stochastic ranking algorithm (Runarsson and Yao, 2000);
- 100% probability of chromosome crossover and a 50% probability of binary gene crossover (known as "uniform" crossover; De Jong, 2006);
- a probability of 1 gene mutation per chromosome;
- elitism in the form of the single best solution from the previous generation being included in the new generation;
- completion of the search after a fixed number of unique solutions have been evaluated; this being set to a 1000 for the unconstrained optimization, and 3000 for the more demanding constrained optimization;
- a population size of 30 individuals with automatic re-initialization of the population if it collapses before completion of the search.

EXAMPLE OPTIMIZATION

The example optimization is based on an atrium of a three-storey commercial building located in Chicago, USA. The atrium is 15m wide by 15m long by 8.2m high with only the southern façade being exposed to the external environment. The other three sides of the atrium are connected to interior spaces that are controlled to have the same thermal conditions as the atrium.

The roof, internal partition walls and the external façade have a light-weight construction; the floor is constructed from uninsulated concrete; and the window cells have a double-glazed construction.

Performance Simulation

The EnergyPlus whole building performance simulation (version 2.0.0.25; Crawley et al. 2001), has been used to evaluate the atrium energy use of each design solution. The atrium has been modelled as an independent zone with the internal partition walls being treated as adiabatic heat transfer surfaces.

The performance of the atrium HVAC system has been modelled using a pseudo-system having an idealized 100% efficiency (the EnergyPlus

'purchased air' model). This model is suitable for estimating the heating and cooling energy use during the early design stages (where there is less focus of the detailed design of the HVAC system; Mourshed et al. 2003). The system is operated 24 hours/day with cooling and heating setpoints for the occupied periods being set to 25.6 °C and 20 °C respectively; night setback setpoint temperatures are 30.0 °C for cooling operation and 15.0 °C for heating operation.

The artificial lighting (and its energy use), is used to supplement daylight illuminance levels by an amount that maintains the illuminance setpoint (500lux), calculated at two reference points (both reference points having equal weight in the control of the artificial lighting). The two reference points are located along the mid-point of the façade width, and at a distance of 25% and 75% of the depth of the atrium.

The building is fully occupied from 09:00 to 17:00; the occupancy is reduced during the periods from 07:00 to 09:00, and 17:00 to 22:00.

Optimization Problem Variables

The southern façade of the atrium has been divided into 15 cells wide by 8 cells high ($nc=15$; $nr=8$; a total of 120 cells and problem variables). Each cell can have one of two possible states (opaque or glazed), and therefore, there are a total of 2^{120} possible solutions to this optimization problem ($1.3x10^{36}$ solutions).

Note that cell ($k=1$, $j=1$), corresponds to the top-east corner cell on the façade and cell ($k=nc$, $j=nr$) to the bottom-west corner cell (expression 8; Figure 2).

Experiments

Four different sets of results have been generated, an **incremental traverse** of the solution space, and three different optimization runs. The incremental traverse is not a formal optimization run but has been used to examine the general characteristics of the optimization problem. In this case, the traverse incrementally adds window cells to the façade (Figure 3) starting from the top-east corner ($k=1$, $j=1$), and ending in the bottom-west corner ($k=nc$, $j=nr$).

Figure 3,
Incremental Traverse of the Solution Space
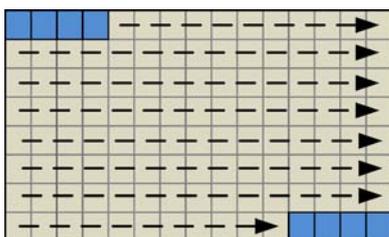
The three optimization runs presented in this paper are:

1. an **unconstrained optimization**;
2. minimization of energy use with the **aspect ratio constrained** to be in the range 1.5 to 1.75, this range being set to allow several alternative window sizes to occur within an 8 cell high façade and having an aspect ratio close to the Golden Ratio (Tables 1); the Golden Ratio (Height/Width=1.62), is often used in defining the geometric proportions of a building, and in this case is set to produce tall-thin windows that emphasize the height of the atrium;

Table 1,
Possible Range of Solutions
with Aspect Ratio Constrained

Height (cells)	Width for an approximate Golden Ratio (cells)	Aspect Ratio (-)
8	5	1.60
7	4	1.75
6	4	1.50
5	3	1.67
3	2	1.50

3. minimization of energy use with the total **number of windows constrained** to be less than or equal to 3.

RESULTS AND ANALYSIS

Figure 4 illustrates that the optimum window area is in the order of 22%, this corresponding to 27 window cells (which in the case of the incremental traverse are located in the top two rows of the façade). Note that, although it is expected that the window cells would be distributed towards the top of the façade (as this maximises daylight penetration and reduces lighting energy use), it is likely that the optimized number and distribution of window cells will be different to that found from the simple incremental traverse of the solution space.

Figure 5, illustrates that the energy use is dominated by the heating of the atrium. However, the optimum window area is dictated by the trade-off between the reduction in lighting (electrical) energy and the increase in cooling energy with the number of window cells. The greatest rate of reduction in lighting (electrical) energy use occurs between 1 and 27 window cells, with no significant reduction for a further increase in number of cells. Although increasing the number of cells further results in a reduction in heating energy use, there is a much greater rate of increase in cooling energy use, this

resulting in an increase in total energy use for more than 27 window cells.
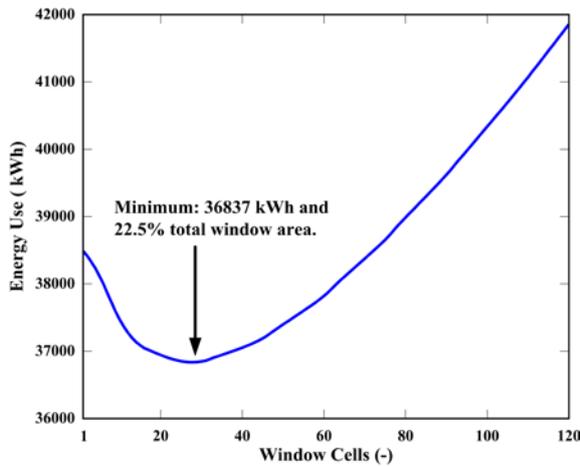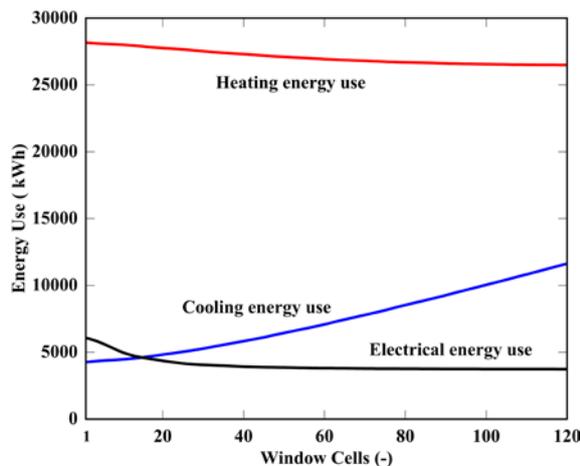
Unconstrained Optimization

Figure 6 illustrates the solution with the minimum energy use resulting from the unconstrained optimization. The total energy use is 36,858 kWh with a glazed area of 21.7% (Table 2), these being similar to the energy use and percentage area found from the incremental traverse (Figure 4). Figure 6 also illustrates that the window cells are biased towards he top-west quadrant of the façade (as indicated by the cross and arrow, the base of the arrow being located in the centre of the façade). However, there is considerable scatter across the façade, which suggests that the optimum energy use is insensitive to the location of individual windows and the percentage window area. This is reflected by the number of solutions in close proximity to the optimum (Figure 7).
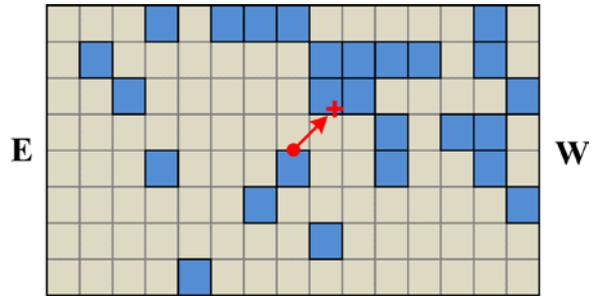
Energy Use (kWh)	Area (%)	Density (%)	Aspect Ratio (-)	Number of Windows
36858	21.7	71	0.71-2.00	8
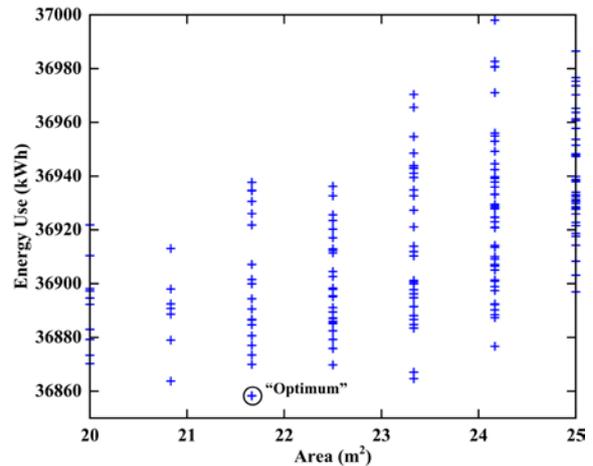
Although a number of alternative solutions are available for inspection and possible use by the designer, they all have the same characteristic that the position of the windows cells are biased towards the top-west quadrant of the façade. The reason for this has been examined by simulating the atrium energy use for a 7 cell wide, by 4 cell high rectangular window located in one of four positions; the top-east corner, the top-west corner, the bottom-east corner, and the bottom-west corner. The window has 28 cells, and occupies 23.3% of the façade area (which is in the region to the perceived optimum area). The impact of window location on energy use is illustrated in Figure 8 (note that the range of values for the vertical energy use axis is the same for all forms of energy use; 500kWh).
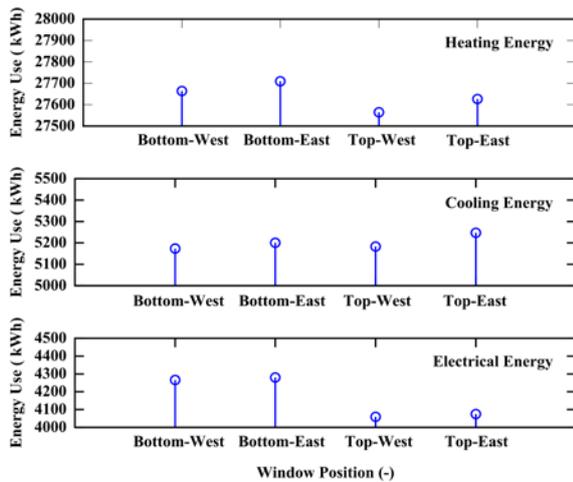
and 10 illustrate the fraction of 500lux required from artificial lighting for the two lighting control (reference) points. In the case of the reference point closest to the façade (Figure 9), positioning the windows at the top of the façade results in an increase in winter lighting load, but a decrease in summer load. The lighting load associated with the control point located furthest from the façade (Figure 10), is reduced in all months, the highest reduction being when the window is positioned in the top-west corner of the façade.

As well as having an impact on lighting energy use and the resulting internal heat loads, the window position has a direct impact on the distribution of beam solar radiation on the internal surfaces of the atrium. In particular, the fraction of total beam radiation falling on the floor is reduced in all months, the greatest reduction occurring during the winter months (Figure 11). A reduction in solar radiation falling on the floor results in a reduction in heating energy use, but an increases in cooling energy use (the *pattern* of heating and cooling energy use with window position evaluated with no lighting load is similar to that in Figure 8, although there is a smaller difference in energy use between "east" and "west" locations). Since, the internal partition walls are modelled as adiabatic surfaces and are thermally light-weight, an increase in the distribution of radiation onto these surfaces is likely to result in an increase in the more "instantaneous" heat gain to the zone, resulting in a reduction in heating load, but increase in cooling load.

Figure 9,
Fractional Lighting Load, Control Reference Point
Closest to the Façade

Figure 11,
Fractional Distribution of Beam Solar Energy to the Floor

Figure 10,
Fractional Lighting Load, Control Reference Point
Furthest from the Façade
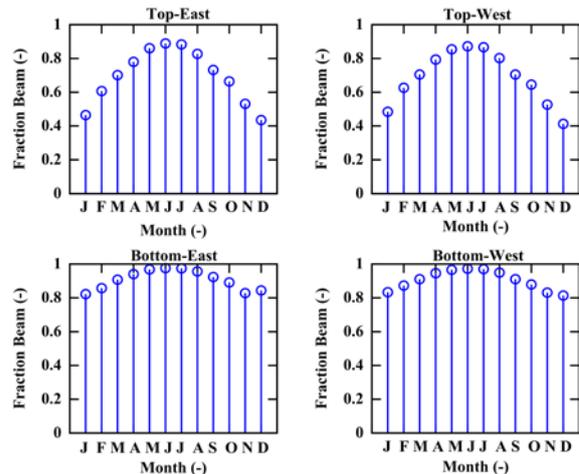
Constrained Aspect Ratio

Figure 12, illustrates the solution with the window aspect ratio constrained to be in the range 1.5 to 1.75. Again, the energy use and percentage windows area are of the same order as the solution found from the incremental traverse and the unconstrained optimization. In this respect, it can be concluded that the optimum energy use has not been restricted by

It is clear that location of the window has the greatest impact on lighting (electrical) energy use. Figures 9

the aspect ratio constraints. However, even though the total energy use is unaffected, the distribution of the window cells across the façade differs significantly from the unconstrained solution. In comparison to the unconstrained solution, the range of aspect ratio is reduced from 0.71 to 2.0 in the unconstrained case, to 1.5 to 1.67 in the constrained case), Tables 2 and 3); the number of windows is also reduced from 8 to 3 (Tables 2 and 3).

The reduction in number of windows is a result of; there being only 5 alternative window geometries that satisfy the aspect ratio constraints (Table 1); and that the total number of window cells being available to form a window of viable aspect ratio is restricted by the optimum window area being in the region of only 22% of the façade. In the case of this solution, two of those geometries appeared in the solution (5 cell high by 3 cells wide, and 6 cells high by 4 cells wide). Given that there are fewer windows, and the percentage areas are similar, it might be expected that the density (compactness) of the windows would be higher for the constrained solutions. However, this is not the case as the unconstrained solution has many single cell windows having a density of 1.0; as a result, the mean density of the windows in the unconstrained solution is 71% (Table 2), compared to only 49% for the constrained solution (Table 3).

Figure 12,
Constrained Aspect Ratio Solution
*(W=west; E=east; viewed from inside the building)*
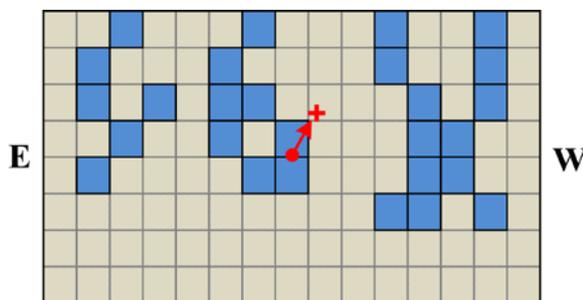
Table 3,
Constrained Aspect Ratio Solution Metrics

Energy Use (kWh)	Area (%)	Density (%)	Aspect Ratio (-)	Number of Windows
36865	22.5	49	1.50-1.67	3

Constrained Number of Windows

Constraining the number of windows to be less than or equal to 3 resulted in a solution having only two windows the location of which is heavily biased towards the top-west quadrant of the façade (Figure 13). This resulted in the solution with the lowest energy use of all experiments (although the

difference in the energy between this solution and that of the other solutions is insignificant); the percentage window area in this case is also slightly lower than for the solutions from other experiments (Tables 2, 3, and 4). The fact that the geometry of the windows in this experiment is unconstrained has resulted in a window having a short-wide aspect ratio, whereas the aspect ratio constraint experiments forced a solution having a tall-thin windows (Tables 3 and 4). Even though the solution found for the aspect ratio constraint experiment would have satisfied the number of windows constraint, allowing the windows to have a "free-form" in this experiment, naturally resulted in denser windows (49% when the aspect ratio was constrained, and 64% when the number of windows was constrained; Tables 3 and 4). Note also that, intuitively, the solution found for the number of windows constrained (Figure 13), appears to be more compact (denser), than that found for the unconstrained optimization (Figure 6). However, this is not reflected in the window density metric, as the unconstrained solution has a higher average density than the constrained number of windows solution (Tables 2 and 4); the reason for this is that there are several single cell windows in the unconstrained, these having the highest density of 1.0.

Figure 13,
Solution with Number of Windows Constrained
*(W=west; E=east; viewed from inside the building)*
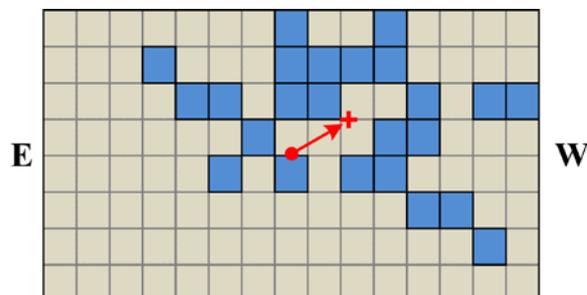
Table 4,
Constrained Number of Windows Solution Metrics

Energy Use (kWh)	Area (%)	Density (%)	Aspect Ratio (-)	Number of Windows
36827	20.0	64	050-0.64	2

DISCUSSION AND CONCLUSIONS

This paper investigates the optimized design of fenestration that is based on the façade of the building being divided into a number of small regularly spaced cells. Each cell on the façade is represented by a discrete problem variable having two possible states (solid construction, or glazed construction). A "window" on the façade is defined

to be a set of adjoining cells. Metrics have been developed that describe the number, position, total percentage area, cellular density, and aspect ratio of such windows. The metrics have been used in the analysis of the design solutions, and in the case of aspect ratio and number of windows, to constrain the form of windows found by the optimization.

Three optimization experiments are described in this paper; an unconstrained minimization of building energy use, and two constrained minimizations of energy use, the first with the window aspect ratio constrained, and the second with the number of windows constrained. In all three cases, the optimization was for the design of a three-storey atrium located in Chicago, USA. Given the binary nature of the problem variables, all three optimization problems have been solved using a binary-encoded Genetic Algorithm (GA).

The results indicate that the GA was able to find near optimum solutions to all three optimization problems. An incremental traverse of the solution space confirmed that both the energy use and percentage window area found by the optimization was near-optimal. The incremental traverse also indicated that although, for this particular example, the magnitude of the building energy use was dominated by heating energy use, it was the rate of change in lighting and cooling energy, and the trade-off between the two, that dictated the optimum percentage window area.

Given that, each optimization experiment resulted in a different distribution of window cells, but that the optimized energy and window area was of the same order of magnitude in each case, it is concluded that, for the example building studied here, the position of the window cells has only a "second-order" effect on energy use. However, in the results from all experiments, the optimized position of the windows cells was biased towards the top-west corner of the façade. Locating the windows towards the top of the façade results in the penetration of daylight to a greater depth in the atrium; correspondingly, this reduces the energy use from artificial lighting, particularly when the windows are positioned towards the top-west quadrant of the façade. The position of the windows also has an impact on the distribution of the beam solar radiation on the internal surfaces of the atrium, which in turn affects heat loads through the different heat loss and storage effects of the various construction elements.

The optimization results also indicate that the design constraints for window aspect ratio and the number of windows are effective in producing different design solutions. The unconstrained optimization resulted in the highest number of "windows" and scatter across the façade. Constraining the aspect ratio of the solutions resulted in fewer windows, but

windows having a more regular geometric shape and which were positioned more evenly across the façade. This effect is a function of the possible number of cells in the façade and the extent to which this limits the number of alternative windows geometries that meet the specified aspect ratio. Constraining the total number of windows (predictably), resulted in a more compact window design. Since the form of windows was unconstrained in this case, the position of the windows was heavily biased towards the top-west corner of the façade.

It can be concluded, that a cellular façade, with the state of each cell (glazed or solid) being optimized by a GA, is not only effective in the minimization of building energy use, but can also result in design solution having innovative and interesting architectural forms. The window metrics developed here, also allow the designer to control the general form of the solutions.

Further research is required to evaluate the performance of the GA, the effectiveness of the metrics in controlling the solutions, the characteristics of the solutions, and the applicability of the approach to real building design.

REFERENCES

Buser, T. 2005. Experiencing Art Around Us, Thomson Wadsworth, Florence, KY, USA.

Caldas, L. and Norford L. 2002. "A design optimization tool based on a genetic algorithm", Automation in Construction, 11, 173-184.

Crawley, D.B., Lawrie, L.K., Winkelmann, F.C., Buhl, W.F., Huang, Y.J., Pedersen, C.O., Strand, R.K., Liesen, R.J., Fisherm, D.E., Witte, M.J., Glazer, J. 2001. "EnergyPlus: creating a new generation building energy simulation program", Energy and Buildings, Vol. 33, No. 4, pp. 319-331.

De Jong K.A. 2006. Evolutionary Computation: A Unified Approach. The MIT Press. ISBN 0-262-04194-4.

Mourshed, M., Kelliher, D., & Keane, M. 2003. "ArDOT: A tool to optimize environmental design of buildings", *Proceedings of IBPSA 2003*.

Runarsson T.P. and Yao X. 2000. Stochastic Ranking for Constrained Evolutionary Optimisation. IEEE Transactions on Evolutionary Computation, 4(3), 284-294.

Wetter M, and Wright J A. 2004. A Comparison of Deterministic and Probabilistic Optimization Algorithms for NonSmooth Simulation-based Optimization, Building and Environment, 39(8), 989-999 ISSN: 0360-132.