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PREDICTING THE TEMPERATURE PROFILE OF INDOOR BUILDINGS BY USING ORTHONORMAL BASIS FUNCTIONS

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ABSTRACT

Orthonormal Basis Functions (OBF) is a structure of dynamic models that have been applied in different classes of dynamic systems. Several works describing the theory and applicability of OBF (Orthonormal Basis Functions) in identification and control can be found in the literature. This work is focused on the problem of finding a Multiple-Input/Single-Output (MISO) OBF model for predicting indoor air temperature and energy consumption. The aim is to analyse an alternative way to do so in relation to well established building energy simulation tools. The model is built in terms of the following variables, that is, the input data, are heating power, outdoor temperature, relative humidity and total solar radiation, and the output data involved is the indoor temperature. The methodology has been tested for the low thermal mass case of the BESTEST model and the output data has been generated by using a building hygrothermal simulation tool. Validation procedures have shown very good agreement in terms of temperature prediction errors between the model and the simulation tool data for both winter and summer periods.

INTRODUCTION

Since gas and oil resources are diminishing this century, the world faces an energy crisis. The current energy problem has a strong connection to global warming and both problems are rooted in the unsustainable use of fossil fuels as our primary energy source. Moreover, when the building design processes are considered in the energy savings concept, HVAC systems are almost criticized as one of the main responsible for the energy consumption in residential and commercial buildings.

Most refrigerants used in air conditioning and refrigeration contribute to global warming in addition to ozone depletion. Even the new non-ozone depleting alternative refrigerants add to the global warming problem. Trying to reduce the HVAC carbon emissions, researches are trying to avoid the energy waste of HVAC systems based on development of building models. In general, the main objectives of obtaining models for thermal analysis in offices, residential buildings and shopping malls are: *i*) estipulate better indoor climate conditions for the occupants; *ii*) avoid the energy waste to decrease the HVAC equipment operating cost and carbon emissions and *iii*) simulate buildings interacting with HVAC equipment.

A classification usually found in the literature for building models is: analytical, semi-empirical and empirical models. The analytical models depend on many parameters and are based on parametric equations. In a similar way, the semi-empirical models depend on test data and less parameters, combining analytical and empirical calculations. Simulation programs such as PowerDomus (Mendes et al., 2003a) and ASTECCA (Mendes et al., 2003b) have analytical and semi-empirical models for HVAC systems.

Finally, empirical models (or black box models) are usually applied to a system when it is viewed in terms of its input, output and transfer characteristics without any knowledge required of its internal workings. Black box models can be extremely useful for a variety of purpose, namely prediction, control, reliability aspects and system management. In hygrothermal systems modeling procedures, when the combination of the HVAC and building envelope study are based on the black box concepts, several detailed analyses can be done, for instance, temperature, thermal comfort and energy consumption predictions (Athienitis et al., 1990; Chen and Chen, 2000). An interesting application is the synthesis of advanced control techniques for HVAC equipments, where a model is usually required for the controller synthesis. An example of an advanced controller in the HVAC context is the Model Predictive Control Technique (Freire et al., 2008b; Donaisky et al., 2007).

Over the last decades, a large number of models have been developed in order to understand building behaviors submitted to different climate conditions. As presented by (Freire et al., 2008a) black box models can be used to verify a building hygrothermal system behavior based on data provided by a building energy simulation software. The models have been used for thermal comfort control based, affecting the indoor temperature relative humidity.

Another example of such kind of modeling has been proposed by Givoni and Krüger. Based on Givoni (1999) method, they presented results of the application of formulae to predict daily indoor temperatures in three monitored low-cost houses in Curitiba, Brazil. In (Papst and Lamberts, 2001), a thermal performance analysis based on Givoni's regression model has been presented. Thermal performance comparisons between three residential buildings have been made. Givoni and Krüger (2003) have presented predictive regression equations to evaluate maximum, average and minimum temperatures of specific houses occupied by different families.

Virk and Loveday (1994) have presented a model in which the multivariable stochastic identification technique is applied to a full-scale test room with dedicated heating, ventilation and air conditioning plant. This model could predict indoor temperature of a test-zone in cold climates.

Considering the wide applicability of black box models found in the literature, the present paper is focused on a methodology, known as system identification, and on the proposal of the use of orthonormal basis functions structure for a HVAC integrated to a building model. The aim is to obtain models capable to describe the HVAC system and building behavior in a structure that could be useful for advanced control law synthesis.

Moreover, this paper can be seen as an evolution of the work presented in (Freire, et al., 2005a) and (Freire, et al., 2008a) where a new black box modeling strategy has been evaluated.

The next section of this paper describes the simulation tool used to generate data for the identification process and the building structure. Then, the building identification method has been presented followed by the model estimation and validation procedures. Finally, an application example of such methodology is discussed and conclusions about this research are addressed.

SIMULATION TOOL AND ENVIRONMENT

The building simulation tool used in the identification procedure is the Brazilian PowerDomus software (Mendes et al., 2003), which is based on a lumped formulation. Its energy balance considers: sensible and latent conductive heat transfer loads; convective heat transfer; long- and short-wave radiation; infiltration; ventilation; loads related to the HVAC system beyond other variables.

Figure 1 shows the user interface of PowerDomus software. It can be seen the BESTEST project geometry used as a building structure in the identification procedures.

PowerDomus is capable to provide all the output reports necessary for the building identification procedures proposed in this paper, becoming data acquisition easier.

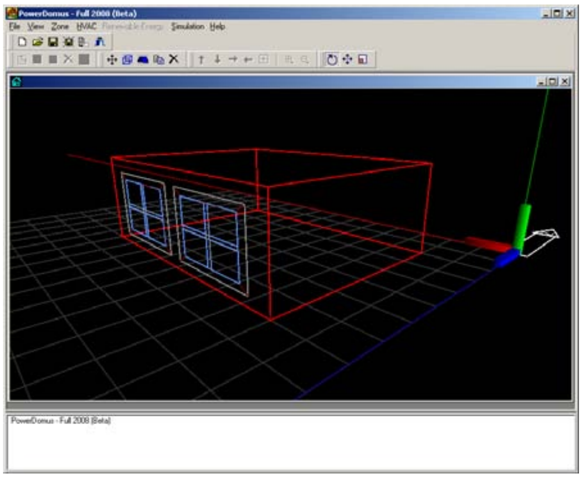

Figure 1. PowerDomus – BESTEST geometry.

The BESTEST building structure has been choosed because of the available data to compare and validate building simulation software (IEA, 1995). PowerDomus has provided results in agreement to different building simulation software (Freire, et al., 2008a). The building structure is defined as follows.

The BESTEST building is based on the single-zone model of the BESTEST methodology for the low thermal mass case (600). Table 1 presents the building envelope material properties. The room dimensions are 8.0m X 6.0m X 2.7m of length, width and height, respectively. There are also two South-oriented 6.0m^2 windows. The input signals applied on the building simulation tool are the Denver’s weather variables, that is, outdoor temperature, outdoor relative humidity and diffuse and direct solar radiations.

The input signals used on the identification procedure are the climate variables described before and the control signal applied to a heating system of 5 kW power. The output signal is the indoor temperature.

IDENTIFICATION METHOD

The system identification is a theory where models are constructed from observed data. In the black box system identification approach, a pair of input/output data is collected from the system and, by means of an optimization procedure, the best model that fits the collected data are computed. When a single pair of input/output data is used, one obtains a SISO

(Single-Input-Single-Output) identification procedure and if more than one input and output data are involved, a MIMO (Multiple-Input-Multiple-Output) procedure is performed.

Table 1
Materials for lightweight BESTEST 600 case.

MATERIAL	k (W/m²)	ρ (kg/m³)	c_p (J/kg.K)	d (m)
Exterior wall (inside to outside)				
Plasterboard	0.16	950	840	0.012
Fiberglass quilt	0.04	12	840	0.066
Wood siding	0.14	530	900	0.009
Floor (inside to outside)				
Timber flooring	0.14	650	1200	0.025
Insulation	0.04	10	1400	1.003
Roof (inside to outside)				
Plasterboard	0.16	950	840	0.0100
Fiberglass quilt	0.04	12	840	0.1118
Wood siding	0.14	530	900	0.0190

A thermal system, as an indoor environment, could be defined as a MIMO (Multiple-Input-Multiple-Output) system. It could be divided in subsystems that relate all the input variables to each output. In the present paper, the considered output is the indoor temperature. The input signals used in the identification process are heating power, outdoor temperature, outdoor relative humidity and total solar radiation.

The MIMO system identification procedure follows the flow depicted in Figure 2 (Ljung, 1999) (Johansson, 1993). This procedure, in the context of the present paper, is discussed as follows.

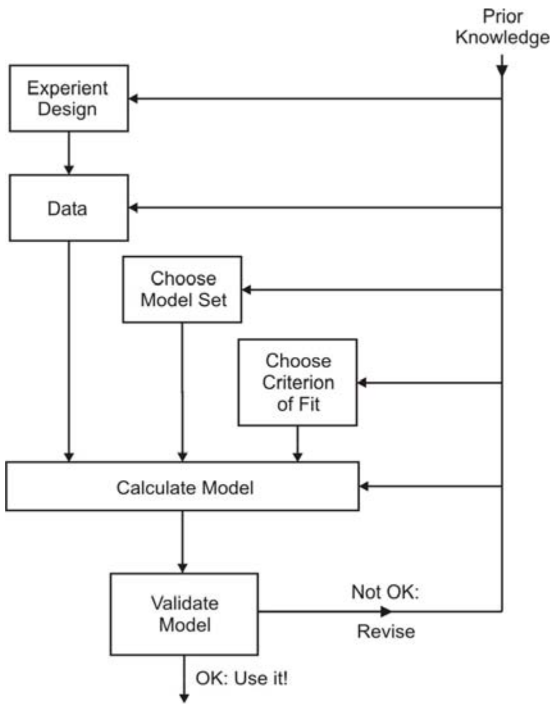

Figure 2. The system identification loop.

Experiment Design and Data

A two-year simulation period has been performed using the PowerDomus software. Results for the last year have been used in the identification procedures in order to reduce the initial conditions effects. A time step of 60*s* and the weather file from Denver city – USA – have been used as input data in the software as well as a random control signal applied to a heating system (a heating system of 5 kW maximum power). The reported data are: the control signal, outdoor temperature, outdoor relative humidity and total solar radiation and the reported output is the indoor temperature.

The experimental data (recorded inputs and outputs over a time interval $1 \leq k \leq N$) for the system identification procedure can be put in the following structure:

$$Z^N = \{u(1), T_{EXT}(1), H_{EXT}(1), S_{EXT}(1), y(1), \ldots, u(N), T_{EXT}(N), H_{EXT}(N), S_{EXT}(N), y(N)\} \quad (1)$$

y(k) is the indoor temperature (in *°C*). $T_{EXT}(k)$, $H_{EXT}(k)$ and $S_{EXT}(k)$ are the outdoor temperature (in *°C*), outdoor relative humidity (in *%*) and total solar radiation (in *W/m²*) respectively. *u(k)* is the input signal applied on the HVAC system that varies from 0 to 1, representing the turn off and turn on states, respectively.

SYSTEM MODELING USING ORTHONORMAL BASIS FUNCTIONS

As mentioned before, the choice of the model structure is an important step on the identification procedures. In (Freire, et al., 2005a) and (Freire, et al., 2008a), the ARMAX structure has been adopted for developing a building coupled to the HVAC model. In present paper, the use GOBF structure in building simulation context is analyzed, so its concepts are reviewed in following.

A SISO linear causal dynamic system, with finite memory, can be described by its transfer function *H(z)* or its impulse response *h(k)*, that is:

$$Y(z) = H(z)U(z) \quad (1)$$

or

$$y(k) = \sum_{i=0}^{\infty} h(k-i)u(i)\,. \quad (2)$$

In this equation, *u(k)* and *y(k)* are the input and output discrete-time signals, respectively. Moreover, if the impulse response has finite memory, *h(k)* can be represented by a series of orthonormal functions, as follows:

$$h(k)=\sum_{i=1}^{\infty} c_i \phi_i(k)\,, \tag{3}$$

where $\phi\, i(k)$, $i=1,\dots,\infty$, is a base of orthonormal functions and c_i is the i-th series coefficient. By substituting (2) into (1):

$$y(k)=\sum_{j=1}^{\infty} c_j \sum_{i=0}^{\infty} \phi_j(k-i)u(k)\,, \tag{4}$$

and by applying the Z-Transform, one obtains:

$$Y(z)=\sum_{j=1}^{\infty} c_j \Phi_j(z)U(z)\,. \tag{5}$$

The series coefficients converge to zero as j increases. So, the series approximation error, defined as:

$$\varepsilon_n(k)=y(k)-\sum_{j=1}^{n} c_j \sum_{i=0}^{\infty} \phi_j(k-i)u(k), \tag{6}$$

can be made as small as wished just by augmenting the series approximation truncation value n. Therefore, the model output $\hat{y}(k)$ is given by:

$$\hat{Y}(z)=\sum_{j=1}^{n} c_j \Phi_j(z)U(Z)\,. \tag{7}$$

The key idea of such approach is to describe the system impulse response, and consequently, the system transfer function, by a series of orthonormal functions. So, an orthonormal basis is defined. Different orthonormal functions can be used in such context and a class of these functions is the one constructed by using the dominant system dynamic(s), given in terms of poles, in a rational transfer function format (Heuberger et al., 2005).

An issue related to the problem of defining a rational orthonormal basis is the choice of the value and number of different modes in the basis functions. For instance, rational orthonormal functions characterized by one single mode, which is given by a real pole or a pair of complex ones, are known as Laguerre and Kautz functions, respectively. However, by defining basis with more than one dynamic response, one can improve the series convergence, leading to a more parsimonious representation in relation to basis having only one mode.

A way of defining basis functions with several modes is to use inner functions, as it will be shown in the following. Such kind of basis is known as Generalized Orthonormal Basis Functions (GOBF) (Van Den Hof et al., 1995) or Orthonormal Basis Generated by Inner Functions (OBGIF). Laguerre and Kautz basis can be viewed as a special case of GOBF/OBGIF ones.

Lets $G_i(z)$ a transfer function with all-pass behavior, defined by:

$$G_i(z)=\frac{1-p_i^{*}z}{p_i z-1}\,, \tag{8}$$

where p_i is a complex pole and $p_i{}^*$ is its complex conjugate. Since $G_i(z)G_i(z^{-1})^*=1$, $G_i(z)$ can be called an inner function. Lets $G_{\mathrm{b}}(z)$ a transfer function given by a cascade realization of n_{b} functions $G_i(z)$, each one having a different pole (real or complex), knowing that the complex ones come in conjugate pairs. These n_{b} poles contain the a priori information about the system dominant dynamics. $G_{\mathrm{b}}(z)$ is also an inner function. Lets the quadruple (A,B,C,D) the space state realization of G_{b}(z), and:

$$V_1(z)=(zI-A)^{-1}B\,. \tag{9}$$

An orthonormal basis can be defined by a cascade realization of n_{g} functions $G_{\mathrm{b}}(z)$ as follows. Lets $\varphi_{ij}(k)$ the i-th element of the signal vector $v_j(k)$ and $V_j(z)$, i.e., the Z inverse transform of $v_j(k)$, given by:

$$V_j(z)=(zI-A)^{-1}BG_{\mathrm{b}}^{j-1}(z), \quad j=1,\dots n_g \tag{10}$$

So $\varphi_{ij}(k)$, $i=1,\dots,\ n_b$ and $j=1,\dots,\ n_g$ is a set of functions that forms a complete basis in the Lebesgue space (Van Den Hof et al., 1995). Equation (7) can thus be rewritten as follows:

$$\hat{Y}(z)=\sum_{j=1}^{n_g} C_j V_j(z)U(Z)\,, \tag{11}$$

where C_j vector contains the n_{b} coefficients of the system orthonormal basis realization. By defining $L_j(z)$ equal to the product $V_j(z)U(z)$, one obtains:

$$\hat{Y}(z)=\sum_{j=1}^{n_g} C_j L_j(z)=\mathbf{C}L(z)\,, \tag{12}$$

where $\mathbf{C}$ and $L(z)$ are the concatenation of C_j and $L_j(z)$, respectively, for j=1,…, n_{g}. The model structure is illustrated by Figure 1 and the space state realization is given by:

$$\begin{cases} L(k+1) &= \mathbf{A}L(k)+\mathbf{B}u(k) \\ \hat{y}(k) &= \mathbf{C}L(k) \end{cases}, \tag{13}$$

where the state vector and matrices **A** e **B** are obtained by using the number of inner functions and its previously selected poles locations.

The procedure for system identification by using OBGIF model structure can be viewed as presented in Figure 1.

i) First, it is necessary to select the basis structure, basically, the function poles (Equation (8)) and the number of functions (n_{g}). These parameters will define matrices **A** and **B** order and elements. *ii*) Second, the model parameters, represented by matrix **C**, are estimated. Such procedure can be performed iteratively and initialized by using some prior knowledge about the system dynamics.

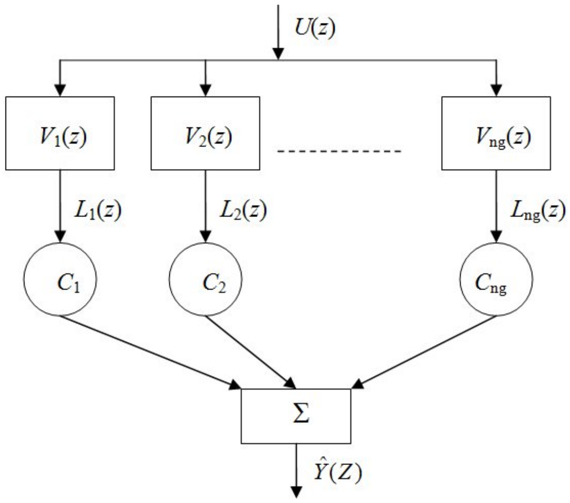

Figure 3. Orthonormal basis model structure.

IDENTIFICATION PROCEDURES

The steps reported by Figure 2 were descibed in the previous section, where the Model Set is represented by the OBF structure. By using the Least Square Criterion to fit the model, in this section, the parameter estimation procedure is presented. It is divided into two identification procedures in order to obtain two building models: *i*) 600FF Winter; and *ii*) 600FF Summer.

For both winter and summer models, the control signal has been turned on for determining the building dynamic in relation to such input.

In the identification procedure presented here, the collected data has a 15-day sample period. The system has 4 inputs and 1 output, i.e., the indoor temperature. The legend for the input signals is shown on Table 1. The MSE (Mean Square Error) criterion is used here to evaluate the fitness of the computed model.

Table 1
Input Data Description.

INPUT LEGEND	INPUT DESCRIPTION
#1 (ºC)	Outdoor Temperature
#2 (-)	Outdoor Relative Humidity
#3 (kW/m²)	Total Solar Radiation
#4 (kW)	Control Signal (HVAC)

600FF Winter

The input data are presented in Figure 4. It represents the data collected for a 15-day winter period in northern hemisphere (from January 1st to January 15th). As presented before, these input signals have been obtained from simulations performed with the PowerDomus software.

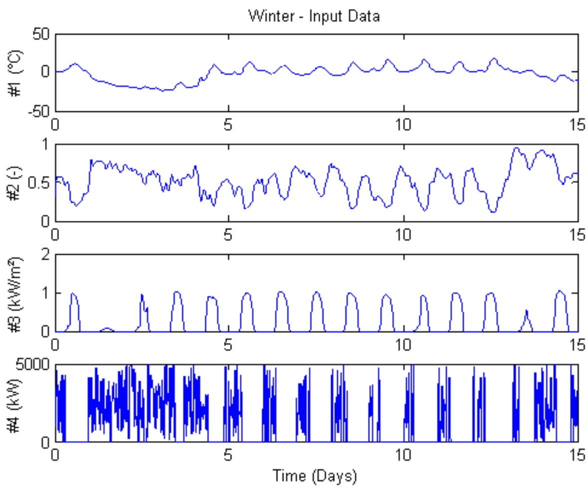

Figure 4. Winter input data for model estimation.

The first step is to define the basis structure, which is given by the set of basis functions poles and by the number of functions in the basis. The basis poles selection procedure is discussed in (Reginato et al. 2007) and the selected poles are given in Table 2.

Table 2 presents the estimation parameters achieved by the algorithm, the system output is shown with the model output in Figure 5, and the MSE between the system output and the model output is 1.3893. It can be notice that, even after 15 days, the model, feeded with the same input data, is able to predict the output with a quit small error.

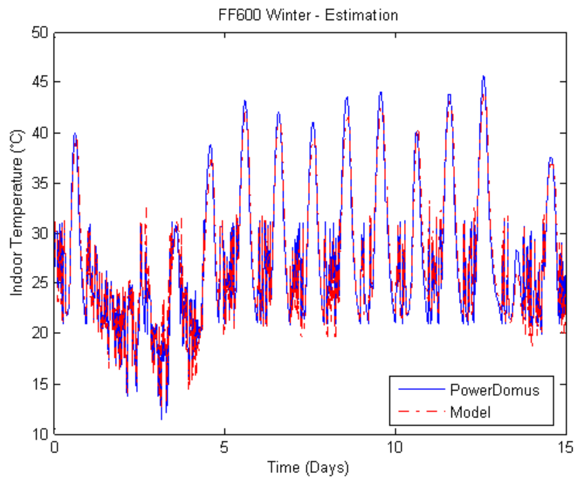

Figure 5. GOBF model estimation for winter season.

Table 2
Winter Estimation Parameters.

INPUT LEGEND	POLES LOCATION	NUMBER OF BASIS
#1 (ºC)	[0.9975 0.9953]	2
#2 (-)	[0.9998 0.9998]	2
#3 (kW/m²)	[0.9929]	1
#4 (kW)	[0.9988 0.8990]	2

600FF Summer

The input data are presented in Figure 6. As seen on that figure, the fourth input signal is zero almost for all the 15 days of the summer period (from July 1st to July 15th), except for the initial samples. This behavior occurs because the environment temperature is high enough, so that the Control Signal actuation (generated by a heating unit) is not necessary.

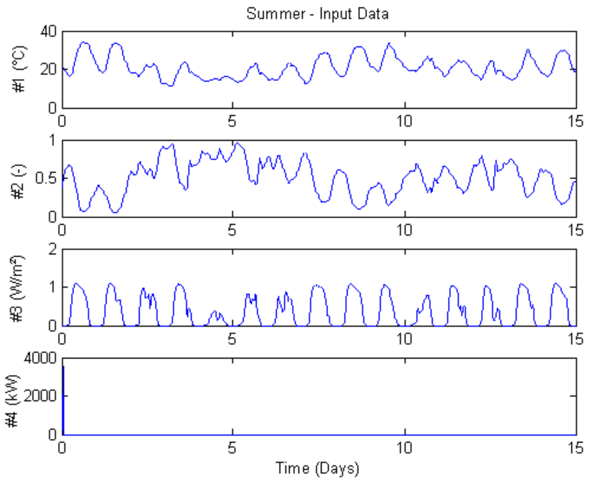

Figure 6. Summer input data for model estimation.

Table 3 presents the estimation parameters achieved by the algorithm, the system output is shown with the model output in Figure 7 and the MSE between the system output and the model output is 1.3308. The mean error and the open loop prediction behavior almost the same for the winter case.

Table 3
Summer Estimation Parameters.

INPUT LEGEND	POLES LOCATION	NUMBER OF BASIS
#1 (ºC)	[0.9996 0.7146]	2
#2 (-)	[0.9995 -0.5556]	2
#3 (kW/m²)	[0.9873 0.9873]	2
#4 (kW)	[0.9796]	1

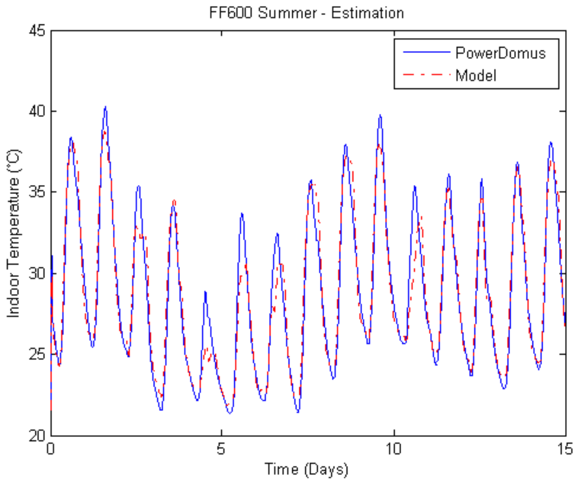

Figure 7. GOBF model estimation for summer season.

MODEL VALIDATION

The validation procedures use the same data previously shown in Figure 4 and 6, except that it does not compute the input signal denoted Control Signal for both seasons. This controlled situation has been performed in order to change the input behavior and consequently its output in order to test the computed models.

600FF Winter

The system and model outputs are shown in Figure 8 and its MSE is 2.1843. Similar to the previous analysis, when the same outdoor model is applied in the model, it is able to predict the indoor temperature several days in advance showing that the dynamic of the building could be captured in representation.

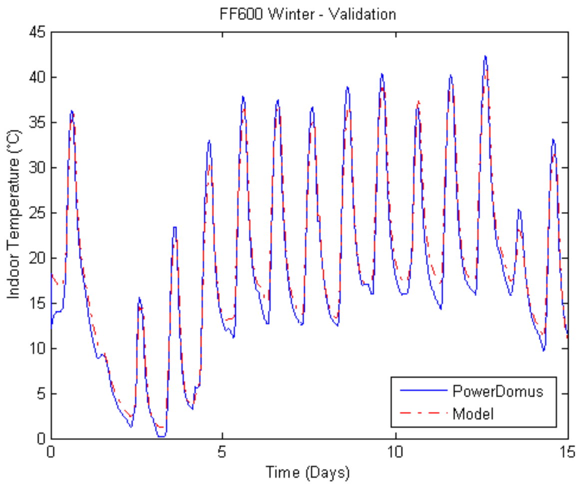

Figure 8. Winter validation curves.

600FF Summer

Figure 9 shows the system and model outputs for the summer season and its MSE of 1.3588. An equivalente bevavior of the previous section can be applied here.

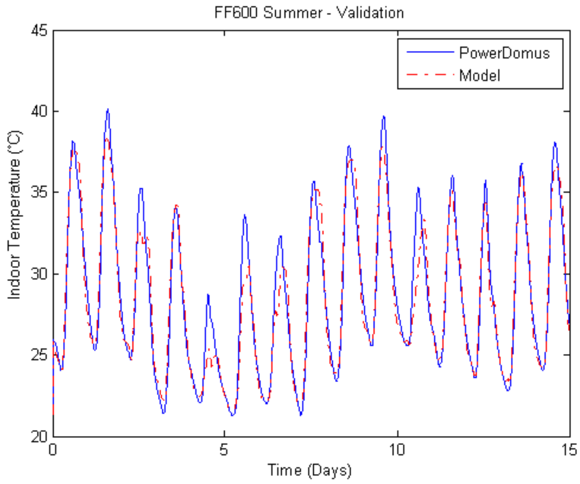

Figure 9. Summer validation curves.

Table 4 summarizes the MSE of all simulations, that is, the estimation and validation cases. It can be see that mean error between the actual data and the model prediction over a future window of fifteen days ahead is close to 1° C, which is acceptable for control and comfort applications. In addition to such small mean square errors, it can be depicted by the figures that the biggest amplitude of errors plots can be estimated as smaller than 3° or 4° C.

Table 4
Models summary indexes

	ESTIMATION	VALIDATION	MEAN
WINTER	1.3893	2.1843	1,7868
SUMMER	1.3308	1.3588	1,3448

CONCLUSION

This paper has described the development of building thermal Orthonormal Basis Functions model based on an identification process. The collected data was obtained from simulations performed with a building simulation tool - PowerDomus.

GOBF models can be adequate to a great variety of purposes, such as thermal comfort and energy consumption prediction, HVAC equipment control and system management, in a rapid and easy way.

In order to test the presented procedure, a Denver meteorological weather file has been used to generate the collected data from the first 15 days of January (winter) and July (summer) using a sampling time of 1 min, providing 21,600 pairs of input/output for the winter period and also the same number for the summer period. In this way, a model developed for the BESTEST 600 building case showed very high correlation coefficients. The heavy mass BESTEST 900 case will be analysed in future works.

For validation purposes, the same weather data has been used to predict the indoor air temperature for the winter and summer periods. The results were compared with those obtained with the same building simulation that created the data for the estimation.

The equations obtained in the identification procedure have shown that it can be used in a very reliable way, providing results as accurate as the original data, which can be gathered from numerical or experimental means. In addition, the equations can be implemented and results in terms of building hygrothermal and energy response can be rapidly obtained for any weather condition, any HVAC capacity and any interior heat gain.

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