

ENERGY EFFICIENCY VS SUBJECTIVE COMFORT: A MULTIOBJECTIVE OPTIMISATION METHOD UNDER UNCERTAINTY

Céline Villa¹, and Raphaël Labayrade¹

¹Université de Lyon, Lyon, F-69003, France.
Ecole Nationale des Travaux Publics de l'Etat,
CNRS, FRE 3237,
Département Génie Civil et Bâtiment,
3, rue Maurice Audin, Vaulx-en-Velin, F-69120, France.

ABSTRACT

A multiobjective optimisation methodology is proposed to optimise building design against both physical performance (such as energy efficiency) and psychophysical performance (such as occupant comfort). The originality of the method is 1) the integration of subjective data obtained from psychophysical tests in multiobjective optimisation method and 2) the management of uncertainties inherent to subjective data through a Monte Carlo process. The methodology was developed through a case study of office lighting optimisation. Psychovisual tests were performed from computer generated images obtained using a light transport simulation package.

INTRODUCTION

The design of a building can be seen as a Multiobjective Optimisation Problem (MOP) that requires to identify optimal sets of Decision Variables (DV) corresponding to the best feasible tradeoffs between conflicting objectives.

Various methods can be employed to solve a MOP. Non-Pareto methods consist in transforming the MOP into single-objective optimisation problem(s) to be able to employ usual techniques and achieve one optimal solution (weighted-sum method, ϵ -constraint method, lexicographic method, etc) (Coello Coello et al., 2007). In order to identify the set of optimal solutions, Multiobjective Optimisation Methods (MOM) are based on Pareto optimality concept. Pareto defined the optimum in multiobjective optimisation by introducing the Pareto dominance notion: X defined with a set of decision variable values dominates another solution X' if and only if X is better than X' for all objectives and there exists, at least, one objective for which X is strictly better than X'. Set of non-dominated solutions are denoted Pareto optimal. The optimal solution set image in objective space is called Pareto front and limits the feasible and the unfeasible tradeoffs space of the objective space (Coello Coello et al., 2007). Figure 1 presents a minimisation problem of two objective functions f_1 and f_2 . C is dominated by A and by B and do not belong to the Pareto Front. A and B are not dominated and are Pareto optimal.

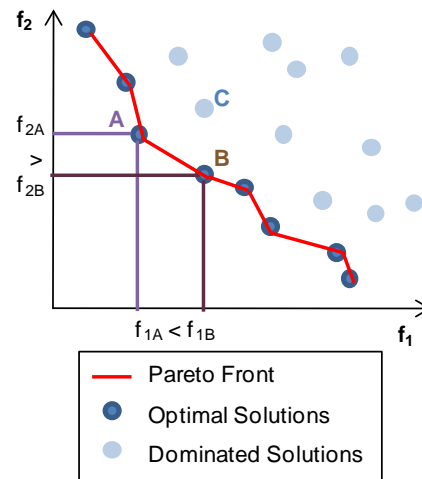


Figure 1 Example of Pareto Front in two-objective space

Among available MOM, Genetic Algorithm (GA) is an evolutionary algorithm based on natural selection and genetic reproduction process (selection, mutation and crossover) which iteratively computes approximate MOP solutions from initial set of random decision variable values (Goldberg, 1989). Candidates are assessed against fitness values to identify best tradeoffs and to converge on a set of approximated Pareto-optimal solutions.

GAs are currently employed to solve building design optimisation problems: in thermal field (Chantrelle et al., 2010) (Hamdy et al., 2009) (Kim and Park, 2009), or for daylighting design problem (Caldas and Norford, 2002) (Manzan and Pinto, 2009) (Wright and Mourshed, 2009). For example, Kim and Park (2009) employed a GA to identify Pareto optimal ventilation system design solutions for three objectives: initial cost, operator cost and consumption, computed with CONTAMW simulation. However, in previous work, the comfort is not taken into account or is computed by means of general physical models (the level of comfort is determined with physical data (e.g. temperature, ...)) non specific with respect to the case study and thus possibly approximate. To avoid such approximations, comfort could be represented in MOM with preferences of future occupants obtained from psychophysical tests relative to the case study.

Integration of human preferences in evolutionary optimisation

Interactive Evolutionary Computation (IEC) was developed to use human evaluation for fitness assignment during optimisation process (Brintrup et al., 2007) (Takagi, 2001) (Takagi, 2009). In the lighting field, Newsham et al. (2004) introduced an IEC method to optimise the comfort in office lighting. They employed a GA with 12 candidates per generation. To solve the mono-objective optimisation problem, one user judges on a rating scale from one to 10 the attraction of 12 lighting solutions displayed one after another. The method was tested with 40 different persons. One optimal solution was found for each participant and was different for each person.

Thus, IEC is suitable if only single-user preferences are taken into account. IEC is currently employed for mono-objective optimisation (Takagi, 2009). Moreover, user fatigue limits the number of psychophysical evaluations of candidate solutions. To manage this problem, the number of generations and the population size are reduced in the evolutionary algorithm at the expense of accuracy (Brintrup et al., 2007) (Newsham et al., 2004). Shibuya et al. (1999) introduced a clustering algorithm in IEC in order to submit to human evaluation only one representative solution of each cluster, thus limiting the number of stimuli.

Work related to the integration of human preferences in Multiobjective Optimisation Method appears in the decision making research area. Proposed methods are aimed at developing decision making tools that allow the decision maker to express preferences about importance of each objective and thus influence the final result (Coello Coello et al., 2007). For example, preferences are taken into account by modifying the optimisation algorithm in order to obtain a local Pareto Front (Branke and Deb, 2004). In (Soebarto and Williamson, 2001), a design tool is introduced based on multicriteria assessment of design solutions compared to a reference building. The authors employed multicriteria decision making techniques in converting the multicriteria problem in a two-criterion problem: each objective is perceived as a benefit value and as a cost value. Then, by computing weighted sum of benefits and costs, design solutions performances are assessed by the designer in the new two-objective space. However, these methods are only available for a decision maker or a small decision making group that already reached a consensus.

As a conclusion, previous work take only into account the evaluation of a single user. In this work, the target is to identify all the tradeoffs that allow to satisfy preferences of panel of future building occupants consisted of more than 30 persons.

To answer the target, a multiobjective optimisation methodology is proposed in the paper to optimise building design by taking into account not only

physical objectives such as energy efficiency but also subjective ones such as comfort. The method proposed to integrate in multiobjective evolutionary optimisation method subjective data obtained from psychophysical tests related to the case study.

Uncertainty management in evolutionary optimisation

Psychophysical data are obtained from statistical tests and are thus inherently uncertain. Thereby, the developed evolutionary optimisation method should take into account this uncertainty.

Uncertainties in the present work are similar to the noise defined in Jin and Branke survey (Jin and Branke, 2005), because values of at least one objective function are uncertain and involve noise in fitness function evaluation. Most of previous methods developed to manage fitness function noise are only available for single-objective optimisation. For more details, reader can refer to Jin and Branke review (Jin and Branke, 2005). In multiobjective optimisation computation, Teich (2001) and Hugues (2001) both propose a modified Pareto-dominance that allow to take into account the Probability Density Function (PDF) of candidate values for fitness evaluation and selection in Evolutionary Computation (EC). However, these methods imply to modify the evolutionary algorithm content. In this paper, the management of uncertainties inherent to subjective functions is handled through a Monte Carlo process (Metropolis and Ulam, 1949) aside evolutionary algorithm.

Content of the paper

The remainder of the paper is organised as follows. In section two, an overview of the methodology is presented. In section three, the case study developed for the application of the method is described. Section four and five are respectively dedicated to the implementation of the two steps of the method: the subjective experiment conducted for the uncertain psychophysical function estimation is presented in section four; the Monte Carlo based optimisation process is presented in section five. Results are discussed in section six. Finally, conclusions and future work are presented in section seven.

OVERVIEW OF THE METHOD

The method is divided in two steps: the estimation of uncertain psychophysical function and the Monte Carlo multiobjective optimisation. An overview of the method is presented in Figure 2.

Step 1: Uncertain psychophysical function estimation

In order to take into account user preferences about the case study being optimised, a psychophysical test is conducted for the assessment of several solutions corresponding to a sampling of the decision variable space.

Then, statistical analysis on collected psychophysical data is carried out to determine global preference of the user sample. For each assessed solution, mean value \bar{x} and standard deviation σ of the sample are computed. Sampling distribution is the distribution of mean value obtained with all possible samples with the same size N from the same population.

The Central Limit Theorem (Howell, 1997) allows to assume that :

- The sample mean value \bar{x} is equal to the mean value of the entire population μ ;
- The standard deviation of the sampling distribution $\sigma_{\bar{x}}$ is equal to σ/\sqrt{N} ;
- The sampling distribution can be approximated by a normal distribution when $N \rightarrow \infty$. In practice, sampling distribution is close to normal distribution for $N \geq 30$.

A sample of more than 30 persons is employed in order to respect the Central Limit Theorem assumptions. Thus, for N higher than 30, the average score of preference and the 0.95 Confidence Interval (95CI), defined with Equation 1, are computed for each assessed solution:

$$95CI = [\mu - t \cdot \sigma_{\bar{x}} ; \mu + t \cdot \sigma_{\bar{x}}] \quad (1)$$

with μ the estimated mean value, t is the value of t-student law with $N-1$ degrees of freedom, N the sample size, $\sigma_{\bar{x}}$ the standard deviation of the normal sampling distribution of samples with size N (from Central Limit Theorem).

Then, the estimation of uncertain psychophysical function is carried out as follows:

- One assessed solution corresponds to one sampled point in decision space ;
- The function value of each sampled point follows the normal PDF $N(\mu, \sigma_{\bar{x}})$ to the related assessed solution;
- The function is estimated throughout the decision space with interpolation method from sampled point function values.

Step 2: Optimisation process under uncertainty with Monte Carlo method

In order to take into account uncertainties on subjective data, a Monte Carlo method is employed for the optimisation process. Monte Carlo method is introduced to carry out multiobjective optimisation for different probable psychophysical functions.

Each Monte Carlo draw corresponds to:

- 1) Random determination of a psychophysical function from random value assignment of each sampled point according to its PDF assessed in step 1;
- 2) A multiobjective optimisation using the randomly drawn psychophysical function as an objective function to optimise;

One draw results in a Pareto Front specific to the used psychophysical function. After n Monte Carlo draws, final result consists of union of specific Pareto Fronts and is called “uncertain Pareto Front”.

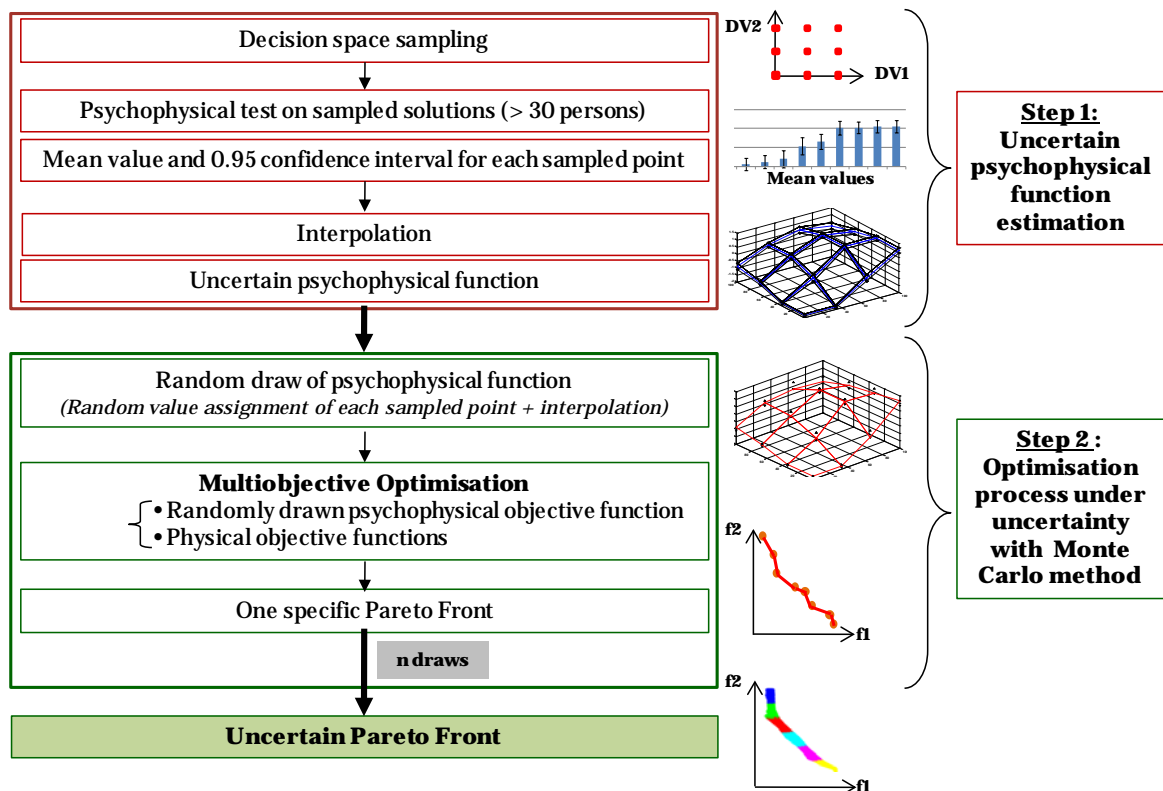


Figure 2 Overview of the method

CASE STUDY

The method was developed through a case study of office lighting optimisation with two decision variables. An office room (2.70 m x 5.70 m x 2.70 m) is lit by two light sources:

- Two recessed luminaries with 28W fluorescent tubes;
- One desk lamp with 11W Compact Fluorescent Lamp (CFL).

Table 1 summarised lamp characteristics.

The case study aims at identifying efficient and suitable indoor luminous environments by varying the luminous flux of the two light sources:

- Between 0 and 5200 lm for ceiling source;
- Between 0 and 900 lm for the desk lamp.

For the method implementation, decision variables are percentages of the maximum luminous flux.

STEP 1: UNCERTAIN PSYCHOPHYSICAL FUNCTION ESTIMATION

Decision space sampling

Lighting design solutions are defined by two decision variables, the percentage of luminous flux of the ceiling source (F_c) and the percentage of luminous flux of the desk lamp (F_d). 16 sampled solutions were assessed, each light source providing 0, 33, 66 or 100% of the maximum flux. Figure 3 presents three of sampled solutions assessed during the subjective experiment.

Psychovisual test

Panel

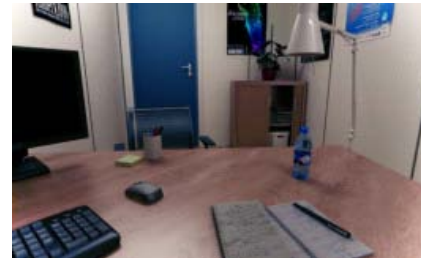
36 persons from the university were involved in the experiment, including 16 students and 20 persons from the staff (either from administration or from research labs). All participants were aged from 19 to 55 years old with 50% of participants wearing glasses. 11 persons were familiar with lighting basics (illuminance, luminance) but only one researcher can be regarded as experienced in lighting design. All participants can be regarded as inexperienced in subjective perceptual studies.

Subjective test protocol

The psychovisual experiment was performed with a virtual reality stereoscopic display device available in the laboratory. The office scene was modelled with the 3DS MAX package. Then 16 renders corresponding to the 16 sampled lighting solutions were computed with the light transport simulation program V-Ray and displayed in three dimensions for assessment. According to a previous study of psychovisual calibration (Villa et al., 2010), V-Ray produces images that cause the closest feeling, regarding luminous environment, to what would be felt in a real-world scene.

Table 1 Lamp characteristics

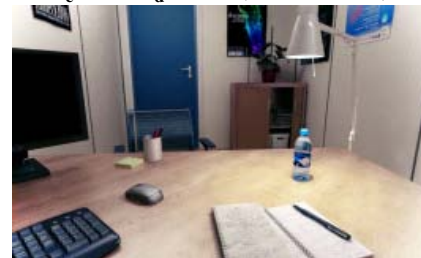
CHARACTERISTICS	CEILING SOURCE	DESK LAMP
Power	28 W	11 W
Maximum Flux	2600 lm * 2 = 5200 lm	900 lm
Luminary Efficiency	0,92	0,81
Colour Temperature	4000 K	3000 K
CRI	80 %	80 %



F_c 66% - F_d 0% (3432 lm/0 lm)



F_c 0% - F_d 66% (0 lm/594 lm)



F_c 66% - F_d 66% (3432 lm/594 lm)

Figure 3 Three sampled solutions assessed during the subjective experiment

Observers were sat at 2.63 meters in front of the screen. The size of images was 1.69 x 1.26 m² and their resolution was 1024 x 768 pixels. The luminance dynamic range of the simulator was 169.4 cd/m² (range 0,6 –170,0 cd/m²).

Images were assessed through paired-comparisons. Observers were asked to imagine themselves working at the desk of the office. They had to choose between image A and image B by answering the question: "Which lighting configuration is the most suitable to work?" Participants could observe as often as needed both images. 120 comparisons were assessed with a different random order for each participant.

Results

To analyse paired-comparison data, Thurstone law of comparative judgment, case V (Thurstone, 1927) was employed to obtain scaled value for each assessed solution. Scaled value defines the average preference

for each solution: the higher the scaled value is, the more suitable the solution for the workplace is. Scaled values, presented in Figure 4, indicate that the most suitable lighting solutions should have a ceiling luminous flux higher than 66% (3432 lm) completed by a switched-on desk lamp. For the maximum ceiling luminous flux, a low level of desk lamp is enough: maximum solutions (F_c 100%, F_d 100%) or (F_c 100%, F_d 66%) do not belong to the most suitable group of solutions (see Figure 4). These results correspond to an average illuminance on the task area between 300 and 570 lux (the EN 12464-1 recommends 500 lux), and a uniformity on the task area higher than 0.57 (the EN 12464-1 recommends 0.7).

Uncertain psychophysical function

Psychovisual test results were used to implement the multiobjective optimisation method. For each assessed solution, scaled value and 0.95 confidence interval allow to define the normal PDF of preference value of each sampled point. The preference values are reported in the decision space at their corresponding position.

A bilinear interpolation method was employed to estimate psychophysical function throughout the decision space in two dimensions (% Ceiling flux F_c , % Desk lamp flux F_d).

In bilinear interpolation, function is composed by triangles defined with three sampled points. Each triangle belongs to a plane defined with Equation 2: a , b , c and d are derived from the coordinates of the triangle vertices.

$$a \cdot X + b \cdot Y + c \cdot Z + d = 0 \quad (2)$$

Then, the preference value $f(X)$ of any solution $X=(x_1, x_2)$ is defined with Equation 3, where a , b , c , d are the parameters of the plane corresponding to the triangle to which the solution belongs.

$$f(x_1, x_2) = (-d - a \cdot x_1 - b \cdot x_2) / c \quad (3)$$

Figure 5 represents the uncertain psychophysical function "suitable office lighting" estimated with bilinear interpolation between sampled points. In dashed line (resp. in dotted line), upper bounds (resp. lower bound) of 95CI of each sampled point are interpolated. A random psychophysical function estimated from random draws of each sampled point values is presented in solid line.

STEP 2: OPTIMISATION PROCESS UNDER UNCERTAINTY WITH MONTE CARLO METHOD

In the second step, uncertain psychophysical function is used as objective function to optimise in the Monte Carlo multiobjective optimisation. Each Monte Carlo draw corresponds to one multiobjective optimisation computation with one probable psychophysical function randomly drawn according to PDF of sampled point preference values. The NSGA-II genetic algorithm was employed for optimisation process (Deb, 2002).

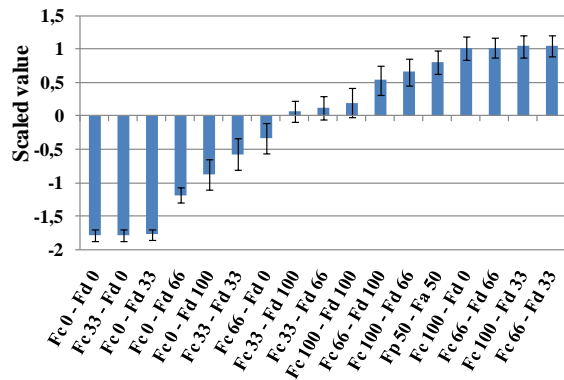


Figure 4 Scaled values and 0.95 confidence interval

Example of "uncertain Pareto Front"

Figure 6 presents results of 10000 Monte Carlo draws for two-objective optimisation:

- The opposite of the psychophysical function "suitable office lighting"¹;
- The consumption C , in Watts, computed according to Equation 4:

$$C = (5200 \cdot \%F_c) / \eta_{\text{ceiling}} + (900 \cdot \%F_d) / \eta_{\text{desk lamp}} \quad (4)$$

with η the luminary luminous efficiency (in lm/W), F_c the percentage of ceiling flux (DV1) and F_d the percentage of desk lamp flux (DV2).

The genetic algorithm NSGA-II was employed with a population size of 200 and 100 generations. In Figure 6(a), result in two-objective space is all optimal solutions called "uncertain Pareto Front" and consists of union of Pareto Front obtained after each Monte Carlo draw. The matched decision variable values of optimal solutions in decision space are presented in Figure 6(b).

Application of "uncertain Pareto Front"

First, the group of identified optimal solutions highlights the interest of using a desk lamp to create an office lighting both energy efficient and suitable. Second, from "uncertain Pareto Front" shape in Figure 6(a), design of lighting installation consuming higher than 40W is not recommended because no enhancement of comfort level is observed. Third, "uncertain Pareto Front" allows to identify suitable ranges of decision variables. For example, optimal lighting installations consuming 40W (i.e. 2.6W/m²) correspond to opposite of psychophysical function values between -0.87 and -1.21 and are defined in decision space with following intervals (Figure 6(b)):

- Between 58 and 65% of maximum ceiling flux (i.e. between 3016 and 3380 lm);
- Between 33 and 66% of maximum desk lamp flux, (i.e. between 297 and 594 lm).

¹ NSGA-II searches for the global minimum of functions. The maximum of the psychophysical function (= the most suitable) corresponds to the minimum of its opposite function.

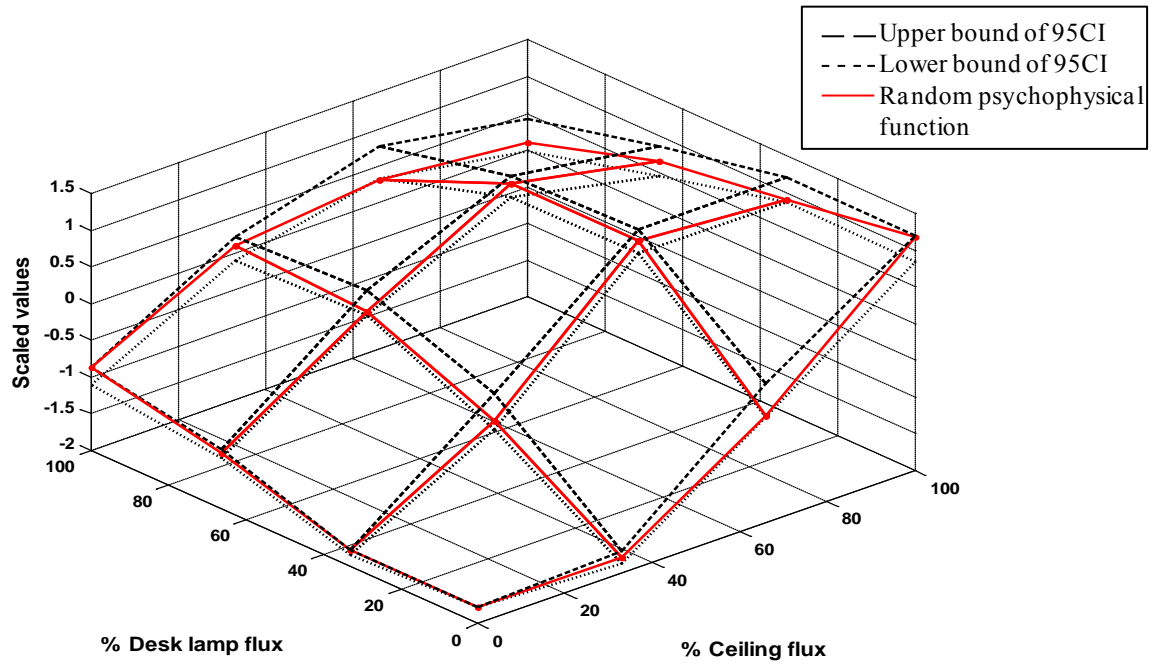


Figure 5 Uncertain psychophysical function "suitable ambient lighting"

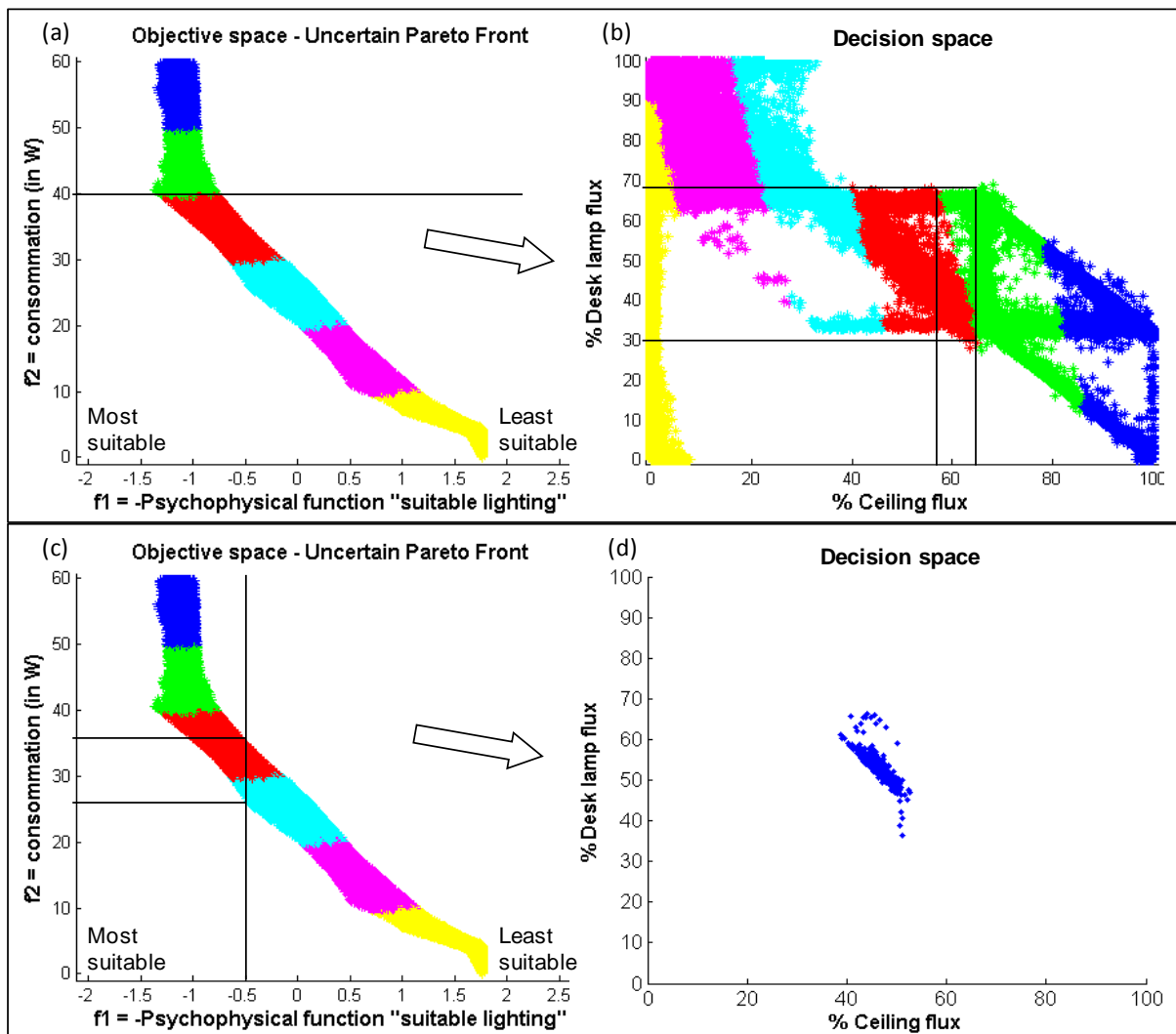


Figure 6 Result of two-objective optimisation for 10000 Monte Carlo draws

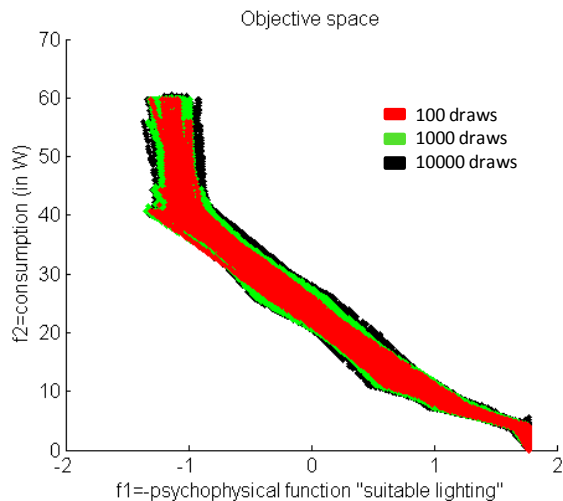


Figure 7 Uncertain Pareto Front for different draw number

Moreover, for constraint values on one or more objectives, optimal tradeoffs range can be identified. For example, to ensure suitable luminous environment for workplace occupants, the comfort objective function threshold could be fixed at -0.5. Thus, from the “uncertain Pareto Front”, the optimal solutions that respect the threshold present:

- Consumption range between 28.8W and 34.5 W (i.e. between 1.87 and 2.24 W/m²) (Figure 6(c));
- DV value range of F_c between 36.4 and 65.5%, F_d between 39.6 et 51.1% (Figure 6(d)).

A lighting installation recommended by EN12464-1 must ensure 500 lux in task area and 0.7 uniformity. The installation defined with ($F_c=100\%$, $F_d=33\%$) provides these characteristics and can be considered as the reference solution. This lighting installation consumes 60W, i.e. 3.9W/m². Installations defined in Figure 6(d) (examples: F_c 36% F_d 50%, or F_c 65% F_d 40%) consume between 1,66 to 2,03 W/m² less than the reference solution with -0.5 comfort level. Thus, by keeping comfortable visual conditions, between 43% and 52% energy saving can be reached compared to the recommended installation.

DISCUSSION

Contrary to Hugues (2001) or Teich (2001), the proposed method deals with uncertainties independently of evolutionary algorithm. As shown in previous section, decision variable range of optimal solutions answering decision maker constraints can be determined from the “uncertain Pareto Front”. As psychophysical function values are uncertain, comfort values of Pareto-optimal solutions obtained from optimisation conducted only with the mean psychophysical objective function would be wrong. At the contrary, with ranges due to uncertainties, the “uncertain Pareto Front” supplies more realistic information on the case study.

Additional results could be obtained by quantifying the risk of not respecting constraints. To that purpose, probability of each optimal solution included in “uncertain Pareto Front” must be known. In future work, probability density of “uncertain Pareto Front” will be computed from PDF of uncertain psychophysical function. Thus, the most robust solutions corresponding to the highest probabilities could be identified.

In addition, each draw takes time. Figure 7 presents results of the two-objective optimisation case study, presented in previous section, for different number of Monte Carlo draws for normal distributions. After 100 draws, the optimal surface is outlined, but as shown in Figure 7, more draws are computed, larger “uncertain Pareto Front” is. 50 hours are required to compute 10000 draws with a population size of 200 and 100 generations using non-optimised Matlab code. Thus, the proposed method is time-consuming. In future work, more efficient numeric schemes will be investigated to reduce computation time.

CONCLUSION

A multiobjective optimisation method taking into account user preferences was introduced to identify best tradeoffs between energy efficiency and comfort. This method, divided in two steps, proposes 1) to estimate a uncertain psychophysical function from psychophysical tests and 2) to perform multiobjective optimisation process against both physical and uncertain psychophysical functions. Psychophysical tests are conducted with more than 30 persons with computer generated images obtained with a light transport simulation package. In order to take into account uncertainties inherent to subjective data, Monte Carlo method is coupled with the optimisation process.

Method result is an optimal surface called “uncertain Pareto Front” that provides range values of optimal tradeoffs for a multiobjective optimisation problem.

The proposed method can be applied on real-world case studies. Office lighting installations that allow to obtain best tradeoffs between energy efficiency and comfort were identified. The use of a desk lamp, even with a low luminous flux, seems to be a good way to achieve both energy efficient and suitable indoor luminance environment to work. Lighting installations ensuring a satisfactory comfort level were identified from the “uncertain Pareto Front” and allow 50% energy saving compared to installation respecting the EN12464-1 standard.

However, limits were identified and will be investigated in future work. Especially, to obtain accurate result, Monte Carlo process requires a large number of draws and thus a large computation time. To improve the method, more efficient numeric schemes will be investigated. Moreover, the method need to be finalised. To that purpose, work about computation of “uncertain Pareto Front” probability density function is in progress and will allow to

supply decision maker with quantified risk of not respecting constraints related to case study under uncertainties.

ACKNOWLEDGEMENT

The authors would like to thanks test participants.

REFERENCES

- Branke, J. and Deb, K. 2004. Integrating User Preferences into Evolutionary Multi-Objective Optimization. Report No. 2004004. Kanpur Genetic Algorithms Laboratory.
- Brintrup, A., Ramsden, J. and Tiwari, A. 2007. Ergonomic chair design by fusing qualitative and quantitative criteria using interactive genetic algorithms. Proceedings on IEEE Transactions on Evolutionary Computing, 12 (3), 343-354.
- Caldas, L.G. and Norford L.K. 2002. A design optimization tool based on a genetic algorithm. Automation in Construction 11, 173-184.
- Chantrelle, F.P., Lahmidi, H., Keilholz, W., Mankibi, M.E., Michel, P. 2010. Development of a multicriteria tool for optimizing the renovation of buildings. Applied Energy 2010, 88, 1386-1394.
- Coello Coello, C.A, Lamont G.B. and Van Veldhuizen D.A. 2007. Evolutionary algorithms for solving multi-objective problems. 2nd ed. Springer.
- Deb, K., Pratap, A., Agarwal, S. and Meyerivan, T. 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II. Report No. 200001. Kanpur Genetic Algorithms Laboratory. IEEE Transactions on Evolutionary Computation, 6 (2), 182-197.
- Goldberg, D.E. 1989. Genetic algorithms in search, optimization and machine learning. Addison Wesley.
- Hamdy, M., Hasan, A. and Siren, K., 2009. Combination of optimization algorithms for a multi-objective building design problem. Proceedings of 11th international IBPSA conference, Glasgow, Scotland, 173-179.
- Howell, D.C. 1997. Statistical Methods for Psychology. Fourth edition. Duxbury.
- Hughes E.J. 2001. Evolutionary Multi-objective Ranking with Uncertainty and Noise. Proceedings of the First International Conference on Evolutionary Multi-Criterion Optimization, Springer-Verlag, 329-343.
- Jin, Y., Branke, J. 2005. Evolutionary Optimization in uncertain environments – a survey. Proceedings of IEEE Transactions on Evolutionary Computing, 9 (3), 303-317.
- Kim, Y.J. and Park, C.S. 2009. Multi-criteria optimal design of residential ventilation systems. Proceedings of 11th international IBPSA conference, Glasgow, Scotland, 1806-1813.
- Manzan, M. and Pinto, F. 2009. Genetic optimization of external shading devices. Proceedings of 11th international IBPSA conference, Glasgow, Scotland, 180-187.
- Metropolis, N., Ulam, S. 1949. The Monte Carlo Method. Journal of the American Statistical Association, 44 (247), 335-341.
- Newsham, G.R.; Marchand, R.G.; Veitch, J.A. 2004. Preferred surfaces luminances in offices, by evolution. Journal of the Illuminating Engineering Society, 33 (1), 14-29
- Shibuya, M., Kita, H. and Kobayashi, S., 1999. Integration of Multi-objective and Interactive Genetic algorithms and its application to Animation Design. Proceedings of IEEE Systems, Man and Cybernetics (SMC), Tokyo, Japan, 3, 646-651.
- Soebarto, V.I and Williamson, T.J., 2001. Multi-criteria assessment of building performance theory and implementation. Building and Environment, 36, 681-690.
- Takagi, H., 2001. Interactive Evolutionary Computation: Fusion of the capabilities of EC optimization and human evaluation. Proceedings of the IEEE 89, 9, 1275-1296.
- Takagi, H., 2009. New IEC Research and Frameworks. Aspects of Soft Computing, Intelligent Robotics and Control, edited by János Fodor and Janusz Kacprzyk, Springer-Verlag, Berlin Heidelberg, 65-78.
- Teich, J. 2001. Pareto-Front exploration with uncertain objectives. Proceedings of the First International Conference on Evolutionary Multi-Criterion Optimization, Springer-Verlag, 314-328.
- Thurstone, L.L., 1927. A law of comparative judgment. Psychological Review, 34, 273-286
- Villa, C., Parent, E. and Labayrade, R., 2010. Calibrating a display device for subjective visual comfort tests: selection of light simulation programs and post-production operations. Proceedings of CIE, Vienna, Austria, 783-791.
- Wright, J. and Mourshed, M. 2009. Geometric optimization of fenestration. Proceedings of 11th international IBPSA conference, Glasgow, Scotland, 920-927.