APPLICATION OF DYNAMIC THERMAL NETWORKS TO THE MODELLING OF FOUNDATION HEAT EXCHANGERS

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ABSTRACT

Foundation Heat Exchangers (FHX) are a novel form of ground heat exchanger for residential applications and, by virtue of lower costs, could increase the uptake of efficient heat pump technology. This work has aimed to develop a new efficient model of such heat exchangers for system simulation.

The recently developed Dynamic Thermal Network approach has been applied to formulate a model of the FHX that includes the basement, pipes and adjacent ground. This response factor approach allows complex three-dimensional geometries such as this to be represented. The formulation of the method and its application to the FHX is described along with a numerical procedure to calculate the required weighting factor series. An improved method of calculating this data and reducing it to a compact form is presented. Experimental data has been used to verify the results.

INTRODUCTION

Reducing energy consumption from new and existing buildings is one of the key challenges for government and industry to meet carbon dioxide (CO_2) emissions reduction targets. For example, around 44% of total UK energy consumption is used by the building sector, which accounts for more than one-third of CO_2 emissions (DECC 2007). A target of 80% reduction in emissions by 2050 has been set in the UK (CIBSE 2004) and in order to achieve this, and move towards to a low carbon soceity, the use of renewable and/or low carbon technologies such as Ground-source Heat Pump Systems (GSHP) is increasingly important.

When installed and sized correctly, GSHP systems can be highly efficient. One of the barriers, however, that still limits the uptake of these systems is the relatively high first cost of drilling boreholes for vertical heat exchangers or, in the case of common residential systems, excavating trenches in the soil for horizontal heat exchangers.

A new type of ground heat exahanger, called a Foundation Heat Exchanger (FHX), has recently been proposed as an alternative to conventional ground heat exchangers for residential applications (Spitler et al., 2010). Instead of digging purpose-made trenches, small diameter ground loop pipes are placed at the outside of the excavations made for the basement or foundations. Supplementary loops may be laid in service pipe trenches in a similar manner. In this way, FHXs can significantly reduce the installation cost, which helps to shorten the payback period and makes it more feasible to apply GSHPs in low cost housing.

Three homes in eastern Tennessee, United States, have installed the FHXs, and were found to work successfully (Shonder et al, 2009). The FHXs in the first house (Christian, 2007) and the more recent experimental houses were designed on a heuristic basis as there is an acknowledge lack of design tools or simulation models. A recent project (see acknowledgements), of which this work forms a part, has sought to address this need.

Xing et al. (2010) has developed two approaches to model the FHX, and these are a simplified analytical model, and a two-dimensional numerical model. The analytical model is based on the principle of superposition of line sources. The numerical model is a two-dimensional finite volume model implemented in the HVACSIM+ simulation environment (Clark, 1985), of which the FHX geometry is represented with a rectangular, non-uniform grid. The numerical model takes approximately seven hours to complete a two-year simulation using a late model desktop PC.

To further investigate the heat transfer between the FHX and a side wall, or a corner of a building, a three-dimension FHX structure may be required. A similar numerical modelling approach such as a three-dimensional finite volume model can be implemented to model the FHX. At the same time, however, the computational cost of running the model would increase significantly, i.e. much longer than seven hours for a two-year simulation. With regard to application in a design procedure, the cost of this process would be impractical.

This research work aims to model the basement and FHX by applying an approach called Dynamic Thermal Networks (DTN), which has been recently proposed by Claesson (2003) and Wentzel (2005). The time-dependent thermal processes are represented as a network to describe the relationship between boundary temperatures and heat fluxes. This network includes a combination of admittive and

transmitted heat paths and time varying conductances that are characterised by series of reponse factors. In contrast to other response factor methods, the response function weightings are derived from an analysis of fluxes found from applying step boundary conditions. The method can be shown to be exact in both continuous and discrete forms and can be applied, in principle, to any combination of multi-layer surfaces of arbitary geometry. Wentzel (2005) demonstrated how this approach could be applied in 'two surface' form to model building walls and foundations, and 'three surface' form to model whole houses. When applying this approach to simulate an FHX, annual calculations can be considerably faster than either a two or three-dimensional numerical model. The general formulation of the DTN approach is described in the next section of this paper.

The step responses used to derive the weighting factors can result in very long series. The sequences of weighting factors required for the DTN calculations can be small for two-dimensional building fabric models. However, for larger ground coupled structures such as the FHX, the sequences can be very long and similarly, the length of the temperature histories that have to be continually updated in the simulation could be very long. To make the computation more efficient and the input data more managable, Wentzel (2005) proposed a weighting factor reduction strategy to reduce the quantities of input data. In this work we have applied this approach but also show how it may be improved for heavyweight structures like FHXs. In this way, the volume of data becomes more manageable, and integration with building simulation software (e.g. EnergyPlus) in a more efficient manner becomes feasible. Wentzel used a hybrid analytical-numerical approach to calculate the step response of large geometrically complex structures. We also show how a purely numerical appoach can be taken that allows automatic and parametric calculation of the weighting factors. The method is verified by reference to experimental data collected from one of the houses in the Tennessee project.

DYNAMIC THERMAL NETWORKS

The Dynamic Thermal Network approach to calculating dynamic conduction heat transfer has a number of advantages that make application to FHXs appealing:

- Arbitrary three-dimensional shapes can be treated as well as simple walls;

- Three or more surfaces with their own boundary conditions can be defined. This is useful where pipes are embedded in the structure.

- A response factor approach is likely to be more computationally efficient than a finite volume numerical method.

- Exact discrete forms for piecewise linear boundary conditions can be derived.

- Numerical models such as finite volume method can be used to derive the response factors for complex shapes.

Although the calculation of the step response can take some effort for a three-dimensional problem, once the weighting factors are found they can be stored for later use in simulations. The weighting factor and temperature history sequences can be very long in some cases and so a data reduction strategy is required. This introduces small errors and makes the calculation non-exact. The significance of these errors is examined later.

The General Formulation

The general formulation of the DTN approach for a three-surface problem is described below. The two-surface formulation is a straightforward reduction of this. The three surfaces can be arbitrary in form and can, in fact, be groups of surfaces to which the same boundary condition applies. A boundary temperature and flux are associated with each surface. This is illustrated with reference to the FHX structure in Figure 1. In this application the first surface consists of the collective basement wall and floor surfaces, the second surface is the ground outside the building and the third surface is the collective pipe inside surfaces.
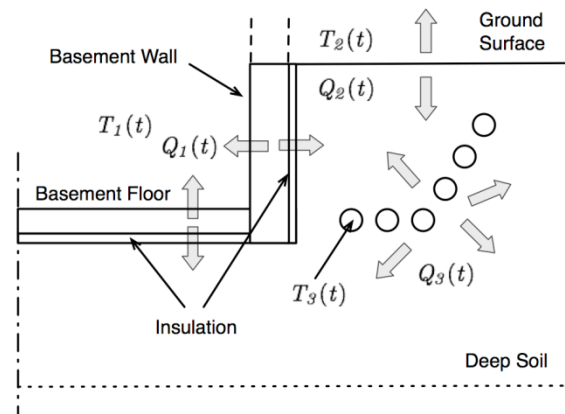

Figure 1 The Foundation Heat Exchanger represented as a three-surface DTN problem (cross-section).

The inside temperature and its heat flux are represented by $T_1(t)$ and $Q_1(t)$ respectively. Similarly, $T_2(t)$ and $Q_2(t)$ represent the outside temperature and its heat flux, and $T_3(t)$ and $Q_3(t)$ represent the pipes temperature and their heat flux. The general form of the Dynamic Thermal Network for a three-surface problem is illustrated in Figure 2.

The temperatures and fluxes are defined at environmental temperature nodes rather than at the surfaces themselves. There are three surface conductances, K_1, K_2 and K_3 and three thermal conductances, K_{12}, K_{13} and K_{23}. Surface conductances are defined at each surface, which are equal to the surface area multiplied by the surface heat transfer coefficients, e.g. K_3

$= A_3 \cdot \alpha_3$. Furthermore, the inverse of the (steady-state) thermal conductances are the thermal resistances between the boundary surfaces, e.g. $R_{13} = 1/K_{13}$.
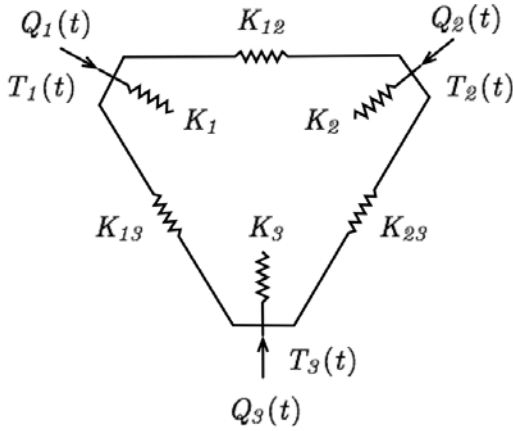

Figure 2 Dynamic thermal network for a three-surface problem.

An important concept is that the boundary heat fluxes Q_1, Q_2 and Q_3 can be divided into two parts: an absorptive heat flux at the surface in question and (in the case of a three-surface problem) two transmittive heat fluxes:

$$Q_1(t) = Q_{1a}(t) + Q_{12}(t) + Q_{13}(t)$$
$$Q_2(t) = Q_{2a}(t) + Q_{21}(t) + Q_{23}(t)$$
$$Q_3(t) = Q_{3a}(t) + Q_{31}(t) + Q_{32}(t) \qquad (1)$$

The dynamic relations between boundary heat fluxes and temperatures for a three-surface problem may be written in the following general way in terms of current boundary and averaged temperatures (Claesson 2003):

$$Q_1(t) = K_1 \cdot \left[T_1(t) - \overline{T}_{1a}(t) \right] + K_{12} \cdot \left[\overline{T}_{1:2}(t) - \overline{T}_{2:1}(t) \right] +$$
$$K_{13} \cdot \left[\overline{T}_{1:3}(t) - \overline{T}_{3:1}(t) \right]$$

$$Q_2(t) = K_2 \cdot \left[T_2(t) - \overline{T}_{2a}(t) \right] + K_{12} \cdot \left[\overline{T}_{2:1}(t) - \overline{T}_{1:2}(t) \right] +$$
$$K_{23} \cdot \left[\overline{T}_{2:3}(t) - \overline{T}_{3:2}(t) \right]$$

$$Q_3(t) = K_3 \cdot \left[T_3(t) - \overline{T}_{3a}(t) \right] + K_{13} \cdot \left[\overline{T}_{3:1}(t) - \overline{T}_{1:3}(t) \right] +$$
$$K_{23} \cdot \left[\overline{T}_{3:2}(t) - \overline{T}_{2:3}(t) \right] \qquad (2)$$

In a three-surface DTN problem, there are three absorptive averaged temperatures ($\overline{T}_{1a}(t)$, $\overline{T}_{2a}(t)$ and $\overline{T}_{3a}(t)$), and six transmittive averaged temperatures ($\overline{T}_{1:2}(t)$, $\overline{T}_{2:1}(t)$, $\overline{T}_{1:3}(t)$, $\overline{T}_{3:1}(t)$, $\overline{T}_{2:3}(t)$ and $\overline{T}_{3:2}(t)$). The exact and general form of the respective temperature differences used in Equation (2) are defined by the current temperature in the case of the absorptive flux, and a set of average temperatures that are defined by weighted temperature histories defined as follows:

$$\left[T_1(t) - \overline{T}_{1a}(t) \right] = T_1(t) - \int_0^\infty \kappa_{1a}(\tau) \cdot T_1(t-\tau) d\tau$$

$$\left[\overline{T}_{1:2}(t) - \overline{T}_{2:1}(t) \right] = \int_0^\infty \kappa_{12}(\tau) \cdot \left[T_1(t-\tau) - T_2(t-\tau) \right] d\tau$$

$$\left[\overline{T}_{1:3}(t) - \overline{T}_{3:1}(t) \right] = \int_0^\infty \kappa_{13}(\tau) \cdot \left[T_1(t-\tau) - T_3(t-\tau) \right] d\tau$$

$$\left[T_2(t) - \overline{T}_{2a}(t) \right] = T_2(t) - \int_0^\infty \kappa_{2a} \cdot T_2(t-\tau) d\tau$$

$$\left[\overline{T}_{2:1}(t) - \overline{T}_{1:2}(t) \right] = \int_0^\infty \kappa_{12}(\tau) \cdot \left[T_2(t-\tau) - T_1(t-\tau) \right] d\tau$$

$$\left[\overline{T}_{2:3}(t) - \overline{T}_{3:2}(t) \right] = \int_0^\infty \kappa_{23}(\tau) \cdot \left[T_2(t-\tau) - T_3(t-\tau) \right] d\tau$$

$$\left[T_3(t) - \overline{T}_{3a}(t) \right] = T_3(t) - \int_0^\infty \kappa_{3a} \cdot T_3(t-\tau) d\tau$$

$$\left[\overline{T}_{3:1}(t) - \overline{T}_{1:3}(t) \right] = \int_0^\infty \kappa_{13}(\tau) \cdot \left[T_3(t-\tau) - T_1(t-\tau) \right] d\tau$$

$$\left[\overline{T}_{3:2}(t) - \overline{T}_{2:3}(t) \right] = \int_0^\infty \kappa_{23}(\tau) \cdot \left[T_3(t-\tau) - T_2(t-\tau) \right] d\tau \qquad (3)$$

It can be shown that Equation (3) reduces to the usual expression for flux in terms of overall conductances and boundary temperatures in the steady state. It also follows that the sum of the weighting factors is always equal to one. Note also that the surface areas are retained in the general heat balance equations and are not normalised.

Step-Response Functions

A convenient way to derive suitable weighting functions ($\kappa_{1a}(\tau)$, $\kappa_{2a}(\tau)$, $\kappa_{3a}(\tau)$, $\kappa_{12}(\tau)$, $\kappa_{13}(\tau)$, and $\kappa_{23}(\tau)$) is to consider the fluxes resulting from step changes in boundary temperatures. All six sets of weighting functions can be found by applying the step boundary condition at one of the surfaces and holding the other boundary temperatures at zero and repeating this for each surface. The form of the transient fluxes resulting from such a step boundary condition are illustrated in Figure 3.

When the step is first applied there is an inrush of heat at the surface that corresponds to the initial absorptive flux and no heat is apparent at the other surfaces. This flux is limited only by the surface resistance and has an initial value equal to K_3 as noted in the figure. As time proceeds the transmitted fluxes at the other two surfaces increase and the absorptive flux diminishes. The absorptive component of the flux is simply the difference between the total flux at the surface and the sum of the fluxes at the other surfaces. The final steady-state fluxes are governed by the overall conductances and the absorptive component of the flux approaches zero as conditions approach this final state.

The weighting functions can be obtained from the step response fluxes and can be shown to be, in general:

$$\kappa_{1a}(\tau) = \frac{-1}{K_1} \cdot \frac{dQ_{1a}(\tau)}{d\tau}, \qquad \kappa_{2a}(\tau) = \frac{-1}{K_2} \cdot \frac{dQ_{2a}(\tau)}{d\tau}$$

$$\kappa_{3a}(\tau) = \frac{-1}{K_3} \cdot \frac{dQ_{3a}(\tau)}{d\tau}, \qquad \kappa_{12}(\tau) = \frac{1}{K_{12}} \cdot \frac{dQ_{12}(\tau)}{d\tau}$$

$$\kappa_{13}(\tau) = \frac{1}{K_{13}} \cdot \frac{dQ_{13}(\tau)}{d\tau}, \qquad \kappa_{23}(\tau) = \frac{1}{K_{23}} \cdot \frac{dQ_{23}(\tau)}{d\tau}$$
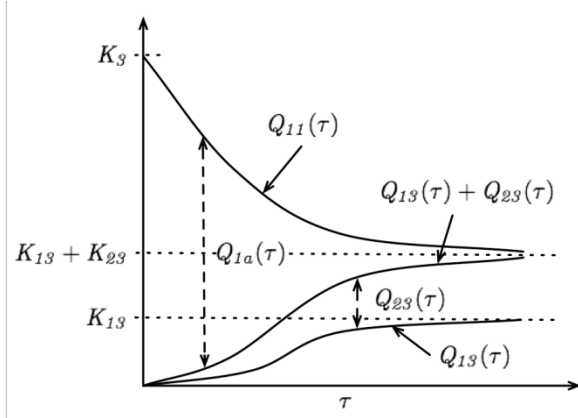
$$(4)$$

Figure 3 Character of the step-response fluxes for surface 3 (the pipe).

Discretization

Claesson (2003) showed that the calculation method could be expressed in discrete form in an exact way for piecewise linear variations in boundary conditions. There are some differences from the continuous form of the equations (Equations (1), (2) and (4)). When the boundary temperatures are defined by a discrete time series, the average temperatures are calculated by the summation of the weighting factor sequence, multiplied by boundary temperature sequences that represent the state at previous time steps. The discrete form of Equation (3) is, given current time step n and sequence of length v:

$$\left[T_{1,n} - \overline{T}_{1a,n} \right] = T_{1,n} - \sum_{\rho=1}^{v} \kappa_{1a,\rho} \cdot T_{1,n-\rho}$$

$$\left[\overline{T}_{1:2,n} - \overline{T}_{2:1,n} \right] = \sum_{\rho=0}^{v} \kappa_{12,\rho} \cdot (T_{1,n-\rho} - T_{2,n-\rho})$$

$$\left[\overline{T}_{1:3,n} - \overline{T}_{3:1,n} \right] = \sum_{\rho=0}^{v} \kappa_{13,\rho} \cdot (T_{1,n-\rho} - T_{3,n-\rho})$$

$$\left[T_{2,n} - \overline{T}_{2a,n} \right] = T_{2,n} - \sum_{\rho=1}^{v} \kappa_{2a,\rho} \cdot T_{2,n-\rho}$$

$$\left[\overline{T}_{2:1,n} - \overline{T}_{1:2,n} \right] = \sum_{\rho=0}^{v} \kappa_{12,\rho} \cdot (T_{2,n-\rho} - T_{1,n-\rho})$$

$$\left[\overline{T}_{2:3,n} - \overline{T}_{3:2,n} \right] = \sum_{\rho=0}^{v} \kappa_{23,\rho} \cdot (T_{2,n-\rho} - T_{3,n-\rho})$$

$$\left[T_{3,n} - \overline{T}_{3a,n} \right] = T_{3,n} - \sum_{\rho=1}^{v} \kappa_{3a,\rho} \cdot T_{3,n-\rho}$$

$$\left[\overline{T}_{3:1,n} - \overline{T}_{1:3,n} \right] = \sum_{\rho=0}^{v} \kappa_{13,\rho} \cdot (T_{3,n-\rho} - T_{1,n-\rho})$$

$$\left[\overline{T}_{3:2,n} - \overline{T}_{2:3,n} \right] = \sum_{\rho=0}^{v} \kappa_{23,\rho} \cdot (T_{3,n-\rho} - T_{2,n-\rho})$$

$$(5)$$

Where the variation of boundary temperatures are in a piecewise linear fashion, the weighting factors can be obtained according to the difference between the flux averages divided by the modified surface ($\overline{K}_1$, $\overline{K}_2$,

or $\overline{K}_3$) or thermal conductance. This is convenient in that the flux history (Equation (4)) does not need to be differentiated. The discrete form of Equation (4) becomes:

$$\kappa_{1a,\rho} = \frac{\overline{Q}_{1a}(\varphi) - \overline{Q}_{1a}(\omega)}{\overline{K}_1}, \quad \kappa_{2a,\rho} = \frac{\overline{Q}_{2a}(\varphi) - \overline{Q}_{2a}(\omega)}{\overline{K}_2}$$

$$\kappa_{3a,\rho} = \frac{\overline{Q}_{3a}(\varphi) - \overline{Q}_{3a}(\omega)}{\overline{K}_3}, \quad \kappa_{12,\rho} = \frac{\overline{Q}_{12}(\varphi) - \overline{Q}_{12}(\omega)}{K_{12}}$$

$$\kappa_{13,\rho} = \frac{\overline{Q}_{13}(\varphi) - \overline{Q}_{13}(\omega)}{K_{13}}, \quad \kappa_{23,\rho} = \frac{\overline{Q}_{23}(\varphi) - \overline{Q}_{23}(\omega)}{K_{23}}$$

$$(6)$$

The time differences are between $\varphi = (vh - h)$, and $\omega = vh$, where h is the time step. The relationship between the step response data and the flux averages at each time step are illustrated in Figure 4 for the FHX structure.
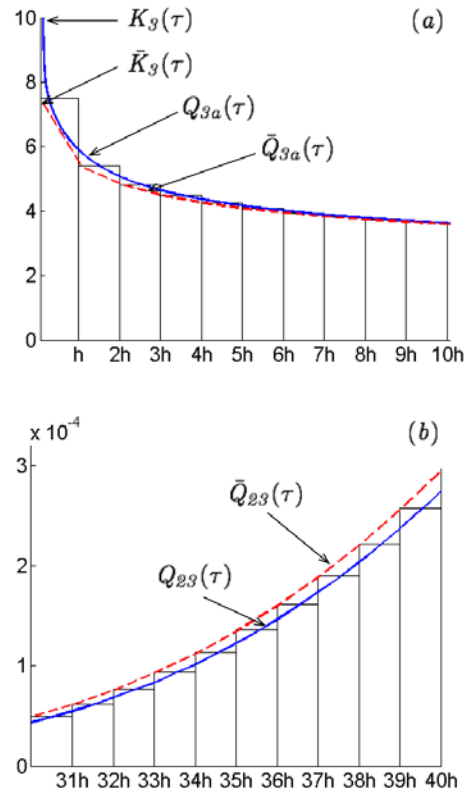

Figure 4 Step-response for (a) absorptive, and (b) transmittive fluxes (solid lines). Flux averages are shown as dotted lines, with bars representing each time step.

NUMERICAL IMPLEMENTATION

The transient fluxes resulting from the application of a step boundary temperature condition are required for the calculation of the weighting factors. These fluxes can be calculated by any convenient method. For multi-layer walls, analytical methods can be employed without much difficulty. Claesson developed an analytical approach for such constructions using a combination of Fourier and Laplacian analytical

methods. For more complex geometries, a numerical method can be used. Wentzel (2005) used a numerical method to derive fluxes for a basement and whole house geometry, for example. We have used an implementation of the Finite Volume Method in the form of the code known as the General Elliptical Multi-block Solver (GEMS3D) to generate the step-response flux histories. The solver applies the finite volume method to solve the general advection-diffusion (convection) equation on three-dimensional boundary fitted grids. Detail description of a similar approach can be found in Ferziger and Peric (2002). The model has been used in a number of ground heat transfer studies (e.g. Deng, 2003). Utilities have been developed to generate meshes of FHX geometries in a parametric manner for this solver.

One of the practical difficulties in using a numerical method to calculate step responses is that, firstly, it is necessary to use a fine mesh near the surface where the step condition is applied, and also very short time steps in order to acurately capture the initial maximum absorptive flux. Capturing this feature of the response essentially requires calculating the temperature gradient near the surface very accurately. Secondly, as many codes use fixed time steps, the number of steps required to reach the steady-state in a high thermal mass structure would be excessive. To address this problem, Wentzel adopted a hybrid approach whereby a generally applicable analytical response was calculated for the initial stage of the calculation. Numerical results with hourly time steps were used for most of the calculation, and another analytical solution was used to calculate the long time scale fluxes. This required some heuristic blending of the numerical and analytical results.

For the sake of automation and practical application in a design or simulation tool, we have developed an entirely numerical approach. We have applied the Finite Volume code GEMS3D and implemented a variable time stepping feature. This allows the calculation of the absorptive flux with very small time steps (of order 0.001 seconds), and the time step size to be gradually increased in a geometric fashion so that the last time steps can be more than one year. To achieve the variable time stepping in an accurate manner, a second order backwards differencing scheme has been adopted. This new formulation allows steps of varying size but retains second order accuracy (Singh and Bhadauria 2009). This approach allows the time steps to be evenly distributed with respect to the log of time.

The general characteristics of the step-response function for the pipe surfaces are shown in Figure 4. The absorpitve fluxes are obtained by subtracting the transmitted fluxes from the total flux by applying Equation (1). It should be noted that, as there is symmetry in the transmitted flux histories, it is only necessary to derive the associated weighting factors once. In total, three step response calculations are required for a three-surface problem.

Weighting Factor Reduction Strategy

The flux history is calculated with varying time step size but the weighting factors have to be found for the selected simulation time step size *h* by interpolating the flux history. The number of preceding temperatures required for the FHX (or basements in general) can be up to a hundred thousand values when using a time step interval of one hour, and a time of a hundred years or more to reach to steady state. These values are used for each simulation time step to calculate the temperature averages needed in the DTN calculation (Equation (2) and (5)).

Flux calculations at each time step using the complete set of weighting factors and corresponding temperature histories can be expensive in computation time. With the sizes of hundred thousand values for the weighting factors and preceding temperatures, a typical two years FHX simulation with time step interval of one hour would require approximately four days to complete the simulation (building walls generally require of the order of one hundred weighting factors and are relatively quick to calculate). To make the computation more efficient, a weighting-factor reduction strategy that aggregates later values and that was developed by Wentzel (2005) has been implemented.

In this approach, the weighting factor series is divided into several sub-series (levels) that have increasing time step size. The procedure is slightly different for the admittive and transmittive components. For the transmittive weighting factors (κ_{12}, κ_{13} and κ_{23}), original intervals (*h*) for each preceding time step are used until the series reaches their maximum. After they decrease below half of the maximum values ($\kappa_{max}/2^1$) the step size is doubled and the weighting factors aggregated accordingly. The second level ($q = 2$) starts after the values have halved again, i.e. $\kappa_{max}/2^2$ with time step length *2h*. The third level ($q = 3$) starts after the values decreased below one fourth of the maximum until they below one eighth of the maximum values i.e. $\kappa_{max}/2^3$ with time step length *4h*. The level reduces according to the power of two in this systematic way, and decreases the number of stored preceding temperature sequences drastically. Criteria are applied to terminate the sequence when the sum of the series closely approaches 1.0. We have implemented this procedure, but found it necessary to adapt it as described later.

FHX RESULTS

The foundation heat exchanger (FHX) has been modelled in both two and three dimensions. Here we only report the two-dimensional results in which case the non-uniform mesh consisted of approximately 19,000 cells. Cell spacing was finer around the FHX pipes, and coarser towards the edge of the soil. The basement wall is 1.59 metres deep under the ground, while the basement slab is 2.5 metres wide and 0.15 metres thick. Dimension of the soil extends 3.5 me-

tres to the right of the basement, and 3.5 metres below. Figure 7 shows the configuration for the experimental house FHX study, and there are six pipes installed under the ground surface. In reality, these continue around the basement perimeter.
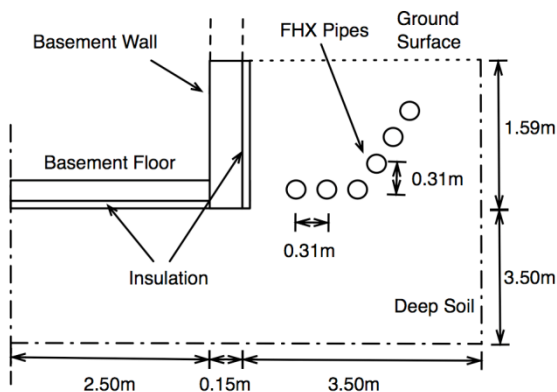

Figure 5 Configuration for the FHX study (not to scale).

Response and Weighting Functions

The nature of the FHX dynamic response and the form of the response and weighting factors can be illustrated by the normalised values using logarithmic time scales. Figure 7 shows the transmittive response functions that are the normalised fluxes (top), and the corresponding weighting functions (bottom). The data is normalised by the conductances K_{12}, K_{13} and K_{23}.

The transmittive flux between the outside and pipe surfaces (Q_{23}), reached its steady-state values after around 5 years. Heat transfer rises rapidly to approximately 20% of the steady-state value after 16 hours and then rises more slowly before reaching its steady-state value. The transmittive fluxes between the inside and pipes surfaces (Q_{13}), as well as transmittive fluxes between inside and outside surfaces (Q_{12}), reach their steady-state values after approximately 6 years. The transmitted heat flow between the basement and the pipes, Q_{13}, starts to rise later (25 hours). Visualisation of the numerical results shows that the nearby soil between room and pipes has to first 'warm up' before heat reaches the pipes. All weighting functions reach their maximum after around 15, 13 and 3 hours for κ_{12}, κ_{13} and κ_{23} respectively.

Figure 8 shows the FHX absorptive fluxes (top), and their corresponded weighting functions (bottom). The response function is normalised by the thermal conductances K_1, K_2 and K_3. The pipe absorptive flux (Q_{3a}) decreases from the beginning (30 milliseconds), and more quickly between around 0.12 and 5.4 seconds. The weighting functions has fallen 50% of its maximum after 2.5 hours. Similarly, the other two absorptive response fluxes (Q_{1a} and Q_{2a}) decrease largely between around 6 minutes and 15 hours. The weighting function has fallen to 50% of their maximum after 5 and 8 hours for κ_{1a} and κ_{2a} respectively.

Values for all these factors become very small later, but still contribute half of the influence for a long period (up to several hundred years).
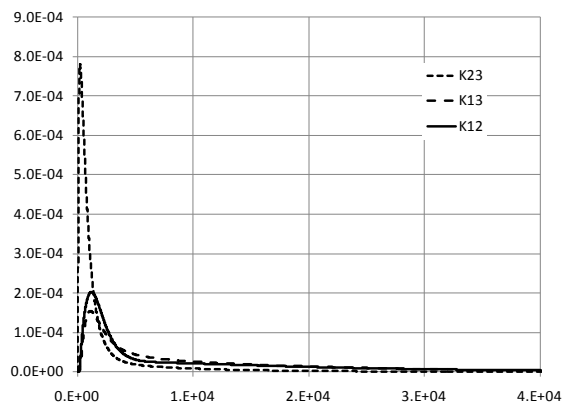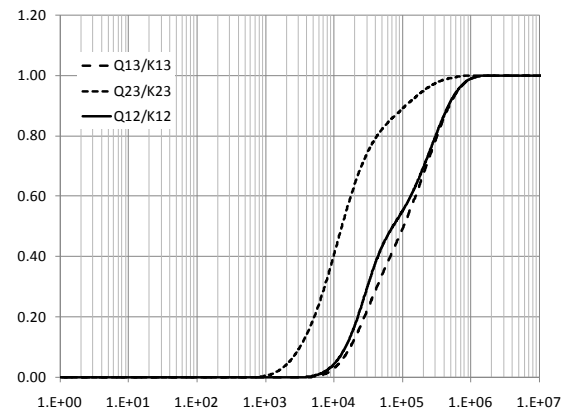

Figure 7 The FHX transmittive response functions (top) and corresponded weighting functions (bottom).The time units are seconds.

Model Verification

To verify the DTN model, comparison is made against the finite volume model, i.e. GEMS3D using the same FHX mesh. The two models are compared in terms of the predicted basement (inside), pipe, and outside heat fluxes. The inside temperature is constant (20 °C), pipes and outside temperatures are taken from the experimental data (Xing et al. 2010). Other parametric studies have shown the basement fluxes and insulation levels can be important (Spitler et al., 2010), but that in the experimental house the insulation makes these insignificant. Figure 9 shows there is acceptably close agreement between the fluxes and the results obtained from GEMS3D. Fluxes predicted for the pipe surface are in good agreement but differences are slightly more significant than the other two surfaces. Since boundary conditions are not periodic and the simulation starts from a constant initial temperature, fluxes are mostly admittive in nature. Furthermore, there is little overall heat transfer from surface to surface in this period. Consequently, the transmittive weighting factors have a weaker influence in this simulation.
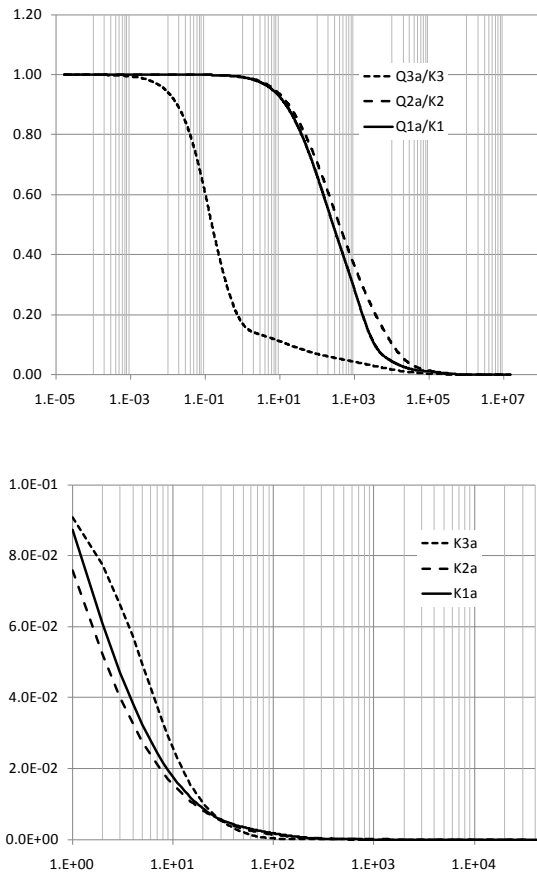
results have been obtained when limiting the number of weighting factors per level to 5.

Figure 8 The absorptive response functions (top) and corresponded weighting functions (bottom) for the FHX study.The time units are seconds.

REDUCTION STRATEGIES

The reduction strategy proposed by Wentzel (2005) aggregated the weighting factors by doubling the timestep size included after the factors have decayed to half of the previous value. Several 'levels' (q) with increasing time step sizes are created as a result of the reduction process. This may often reduce the weighting factors required by two orders of magnitude. However, testing has shown that for the FHX geometry the number of weighting factors (κ_{1a}, κ_{2a}, κ_{3a}, κ_{12}, κ_{13} and κ_{23},) can still amount to more than one hundred thousand values even after the reduction process. This is because the fluxes decay very slowly towards the steady-state value. The quantity of data that would need to be transferred to a building simulation program would consequently be unmanageable, and too slow for calculations.

In order to improve this situation a more aggressive reduction process has been implemented. In this procedure the number of weighting factors at each level of reduction is strictly limited. After a certain number of factors at a given level have been calculated the time step is doubled. This occurs independently of whether the weighting factor values have fallen to half of the previous value. A range of criteria controlling the initial level, tolerances and number of factors per level have been tested. Good
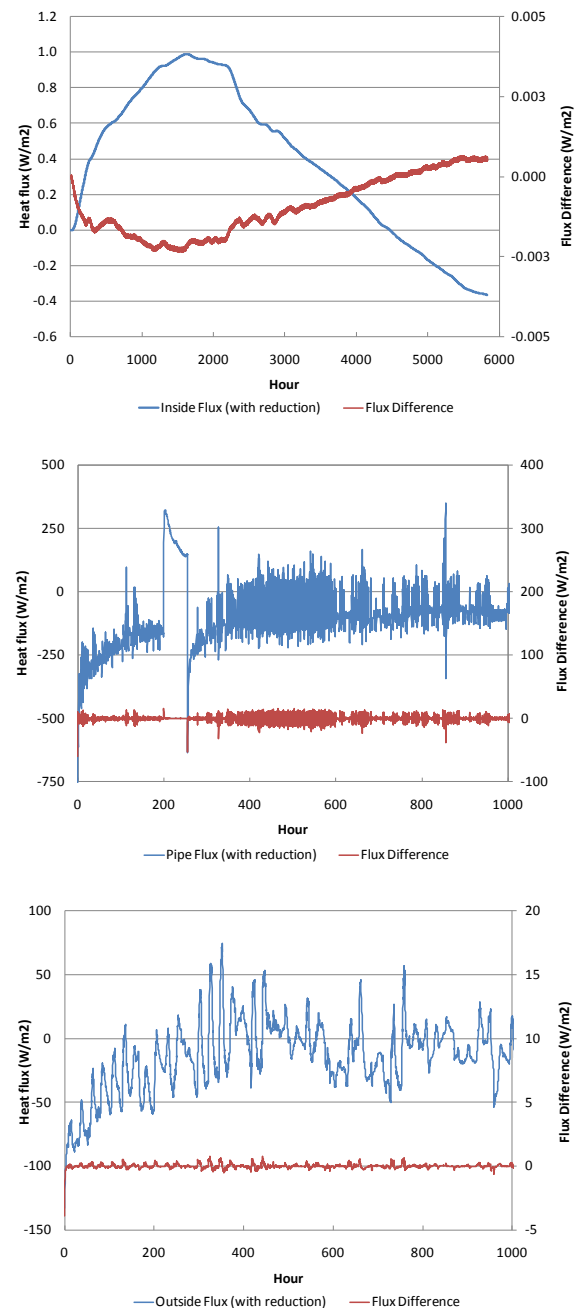
Figure 9 Inside (top), pipe (middle) and external (bottom) surface heat fluxes predicted by the DTN model and compared with the corresponding GEMS3D model results.

The unreduced (CASE1), original (CASE2) and more aggressive reduction method (CASE3) are compared in Table 2. The first group of weighting factors, particularly the absorptive factors, cannot be reduced without compromising the calculation of the short term responses and so it appears that the number of factors cannot be reduced any less than about two hundred in total.

Reducing the weighting factors and aggregating the effect of previous temperatures can potentially affect

the accuracy of simulations. To test the effect of the reduction a trial simulation has been carried out to examine the FHX pipe heat flux using the experimental inlet temperature data. The results are shown in Figure 10. The resulting pipe fluxes with the original and more aggressive reduction method agree very closely. The difference is shown against the secondary axis and amounts to less than one percent difference in predicted flux. It should also be noted that the response to the relatively high frequency fluctuations in fluid temperatures is mostly governed by the absorptive weighting factors of the pipe surface. These have not been reduced as significantly as the transmittive factors.

Table 2
A comparison of the quantity of weighting factors according to reduction procedure along with the corresponding simulation time.

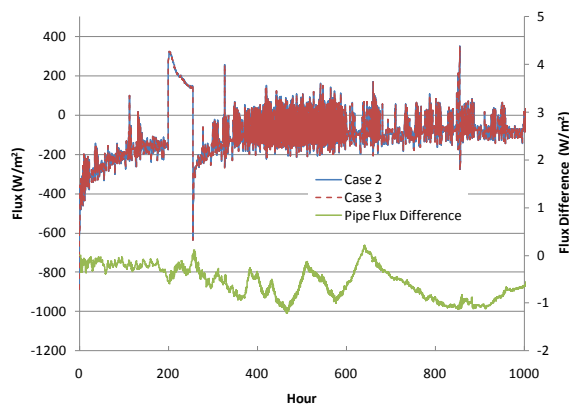|	CASE1	CASE2	CASE 3
Reduction	No	Wentzel	Aggressive
Total number of κ_{1a}	1e6	61812	179
Total number of κ_{12}	1e6	70211	221
Total number of κ_{3a}	1e6	77	71
Total number of κ_{12}	1e6	34145	76
Total number of κ_{13}	1e6	46758	78
Total number of κ_{23}	1e6	9571	87
Time to complete	7.6hrs	29mins	4.47secs

Figure 10 Pipe surface heat flux predicted by the DTN model with different degrees of weighting factor reduction. Case 2 and Case 3 are defined in Table 2.

CONCLUSIONS

Foundation Heat Exchangers (FHX) are a novel and cost effective form of ground heat exchanger for residential applications. This work has aimed to develop a new efficient model of such heat exchangers for system simulation.

The recently developed Dynamic Thermal Network approach has been applied to formulate a model of the FHX that includes the basement, adjacent ground and pipe surfaces. A method to fully generate the required weighting factors by a numerical method has been described and tested. Aggressive reduction

of the weighting factors has resulted in a model that is both able to accommodate complex geometries, and calculate transient heat transfer in a computationally efficient manner. Further work is being undertaken to further validate the model using experimental data and to incorporate the model in a whole building simulation environment.

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