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THE APPLICATION OF STATISTICAL CONCEPTS TO THE WIND LOADING OF STRUCTURES

by

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For written discussion

SYNOPSIS

In the Paper the statistical concepts of the stationary time series are used to determine the response of a simple structure to a turbulent, gusty wind. This enables the peak stresses, accelerations, deflexions, etc., to be expressed in terms of the mean wind velocity, the spectrum of gustiness, and the mechanical and aerodynamic properties of the structure. In this connexion it is pointed out that the resistance in fluctuating flow may be significantly greater than that in steady flow, such as that prevailing in most wind-tunnel tests. An expression for the spectrum of gustiness near the ground is given, which takes into account its variation with mean wind velocity, roughness of the terrain, and the height above ground level. The statistical distribution of peak values over a large number of years is related to the statistical distribution of mean values by means of a so-called "gust factor". A map defining the climate of extreme hourly wind speeds over the British Isles is provided. In association with the gust factor, this enables predictions of extreme peak wind loads with any given return period to be made.

As an illustrative example, the wind loads which might arise on a flexible arc-lamp standard and a rigid water tower are compared. This emphasizes the considerable differences which may arise in the effective wind loads owing to differences in the local ground roughness and the dynamic characteristics of the structure.

INTRODUCTION

IT IS now a well-established principle in the design of almost all above-ground structures to make some allowance for wind pressures. For exceptionally exposed structures such as skyscrapers, tall masts, long-span bridges, radio telescopes, etc., it may be a major consideration governing the initial cost and eventually the inherent safety of the structure.

2. Traditionally these pressures were calculated on the assumption that the fluctuations in the velocity owing to gusts could safely be disregarded and that the velocity could be taken as invariant with time and space. This simplification was convenient in that pressures could then be regarded as static and determined from simple wind-tunnel experiments on models in a steady airstream. From

The anticipated revenue for project purposes is therefore based on the anticipated cropping and a tentative scale of charges per acre, determined by the assessment system adopted. A 5% deduction is usually made to cover shortage due to failures.

The railway, posts, telegraph and telephone, and other "commercial" departments will forecast their revenues in accordance with their own rules.

The other departments, financed from the tax revenue of the State, will report their anticipated costs to finance department. It is the duty of the finance department to estimate the tax revenue expected from the Colony, and to report to Government with their recommendations.

The final project report and financial estimate is usually a considerable volume, replete with maps, plans, tables, and graphs, and embraces all the estimates by the interested departments. If the project is shown to be remunerative, there should be no doubt about sanction.

Conclusion

As design progresses, it will be necessary to revise previous assumptions and modify decisions. Patience and tenacity are required to reach the conclusion in the form of a coherent and co-ordinated project.

Yet another quality is essential to success. The engineers must have a robust faith in the undertaking. Even the most successful irrigation schemes have met with uninstructed, interested, or even perverted criticism which can only be met by meticulous care at every stage of preparation of the project.

these experiments certain aerodynamic coefficients, such as the coefficients of pressure and drag, were found which were assumed to apply to the model and its prototype alike. The only further information needed to calculate the pressures was a suitable value for the "design wind velocity". Since neither the maximum velocity of the wind nor the effects of sudden gusts could be predicted with any certainty, it was common practice—and still is frequently—to use the highest instantaneous velocity recorded by some nearby anemometer. This, it was believed, would at least be a conservative estimate.

3. Although this traditional procedure was simple and depended on data which could be easily obtained, it was soon realized that it was not ideal. Some allowance might for instance be made for the apparently transient and localized nature of gusts. This possibility was investigated in turn by Baker[1] in 1884, Stanton[2] in 1925, and Bailey and Vincent[3] in 1939. Although their experiments suggested that some alleviation might be expected from this source no reduction was in fact recommended. From this point of view, therefore, their results tended to be inconclusive, largely through the lack of a suitable statistical framework into which to fit their observations. A greater degree of success was achieved by Sherlock, who was also concerned with another problem. This was that the indicated "maximum gust velocity" used in design depended largely on the response characteristics of the particular instrument. This could lead to the somewhat illogical situation in which design wind loads were higher where the nearby anemometer was less sluggish or even better lubricated.

4. As an alternative to the practice then current, Sherlock[4] in 1947, advocated the use of an average instead of an instantaneous velocity, together with certain "gust factors" which would allow for the additional effects of gusts. A 5-min averaging period was suggested for the former since climatological records for this period were available in the United States. In deciding suitable gust factors, Sherlock inferred from measurements of the build-up of lift forces on an aerofoil penetrating a sharp-edged gust that, analogously, a gust must traverse eight or ten diameters of an object before the full drag pressures would be felt. On this basis, for example, he concluded that small structures, such as houses, would not respond to gusts lasting less than about 2 sec. For larger structures the period would be correspondingly longer.

5. From detailed measurements of wind velocity made on a tall tower during several winter storms, Sherlock was able to determine the ratio of the "most probable" 2-sec mean velocity to the simultaneous 5-min mean velocity. This was the so-called "2-sec gust factor". The gust factors for other intervals were also determined. In each case they were found to decrease with height above the ground, a feature also noted by Deacon[5] in 1955, who investigated a somewhat greater height range.

6. Although this approach undoubtedly represented a notable advance in the understanding of wind loading, and in particular of the effect of gusts, it still possessed serious limitations. First, the results could only be expected to apply to open country sites similar to that used in the investigations; at much rougher sites, such as the centre of a large city, the gustiness would almost certainly be more intense. Secondly, the approach fails to take account of the history of the loading pattern. It is, for example, incapable of predicting either the consequences of a sequence of gusts striking the structure or the likelihood

[1] The references are given on p. 471.

of such an event occurring. For flexible structures such as tall towers and long-span bridges, this could be a source of serious dynamic stress magnification which no theory based on purely statical assumptions could allow for.

7. Parallel to these investigations into the effect of gusts, other aspects of the problem were under review. The increase of mean wind speed with height, for example, was studied by engineers and meteorologists alike and their findings were reflected in the wind-load requirements of most design codes. The practice of basing design wind velocities on the maximum velocity on record (whether this was an average or instantaneous value), was questioned. This, it was pointed out, made no allowance for the number of years records had been kept and placed too much reliance on the instrument operating satisfactorily on the one isolated occasion when the severest winds were encountered. An alternative was to use the statistical theory of extreme values which, it was shown, enabled the average number of years between recurrences of any specified wind velocity to be predicted. Design winds could then be based on the annual maximum for a number of years instead of on the one critical measurement. In addition, the recurrence period could be chosen in keeping with the expected lifetime of the structure and an appropriately smaller design wind velocity could be used for more temporary structures.

8. However, in discussions of both the variation of wind speed with height, and the estimation of extreme wind velocities, no account was taken of the important part played by surface roughness, the effect of this being to retard the mean wind velocity near the ground in rougher regions, so causing it also to increase with height more rapidly and to increase in gustiness. Wind-load requirements which were generally specified as being typical of open country conditions (although frequently derived from data which were not) therefore tended, in cities, to overestimate the effect of the mean wind velocity and to underestimate the effect of gusts. The total effect clearly depended on the flexibility of the structure and its susceptibility to fluctuating loads.

9. As this discussion suggests, most of these earlier developments in the formulation of wind loads were concerned with the meteorological and climatological aspects of the problem. The terms of reference were still the traditional ones. That is to say the aim was to represent the turbulent gusty wind by an equivalent steady velocity which could then be translated into the pressures acting on a structure by means of simple wind-tunnel model tests in a steady airflow.

10. Although it has been generally assumed that scale effects are negligible for the bluff, unstreamlined shapes which are representative of most civil engineering structures, it is worth while to consider the general conditions for similarity between the pressures on a model and on the prototype structure. Principally they call for similarity in: (i) the shape, (ii) the Reynolds number, and (iii) the kinetic properties of the incident flow. Earlier it was believed that their relative importance was in the order given; that is to say, similarity of Reynolds number was more important than careful simulation of the velocity profile and the turbulent character of the flow. Jensen[6] has pointed out, however, that exactly the reverse may be true. Since the wind is fully developed turbulent flow in the earth's boundary layer, disturbances produced in it are independent of the Reynolds number of the roughness elements or the obstacles placed in it. (This principle has been clearly illustrated by Nikuradse's pipe-flow experiments

described in textbooks on hydraulics.) What is important therefore is to maintain similarity of the approaching flows. Since this is governed by the so-called roughness length of the surface, Jensen concludes that the correct model law requires that the roughness lengths in the model test be to the same scale as the model itself.

11. Commenting on the considerable number of wind-tunnel tests which have been carried out over the period of years, Jensen observes:

> "These investigations . . . are to some extent misleading because the test procedure, especially the model law, has been wrong. It may seem strange that within a vast research field incorrect model-laws have been applied, but the explanation is simple and not very flattering; the model tests have practically never been checked by full-scale tests in nature."

12. In fact the results of the two or three full-scale tests which had been conducted were generally at variance with wind-tunnel tests (see Bailey[7], 1933; Rathbun[8], 1940; Kamei[9], 1955).

13. Nevertheless, provided the correct model law is used, wind-tunnel tests are still a highly valuable tool for determining the average pressures on a structure. Whether or not it is possible to determine the fluctuating pressures due to gustiness by the same means is still not certain, and depends on whether or not the structure of the turbulence in the wind tunnel is similar to that in the natural wind over the significant part of the spectrum.

14. This discussion has so far attempted to summarize the developments that have taken place in the approach to the wind-loading problem. Several gaps still appear to exist in the chain of reasoning whereby predictions of the distress caused in a structure by the wind are made from general information on the climate and nature of the wind. Partly, this has been due to a lack of the necessary basic information and partly to the intractable methods used to solve some aspects of the problem—the effects of gusts for instance. In addition, important considerations such as the effects of ground roughness and the possibilities of serious dynamic stress magnification have not been allowed for.

15. The Paper attempts to integrate the various aspects of the problem into a comprehensive theory and to present the data needed to apply it. The approach makes use of the concepts of the stationary time series which have already been applied successfully to the problems of the gust-loading of aircraft (Liepmann[10], 1952). The final aim has been to predict the probability of damage to a structure owing to high winds. The question of what probabilities are acceptable lies outside the province of the Paper. Suffice it to say, however, that the answer depends on a large number of factors, such as the risk to life and property, the cost of replacement or repairs, insurability, and public opinion. The probability level of the loads themselves will depend on whether they represent "proof" or "ultimate" loads, that is to say on the design method (see Horne[11], 1950). In practice the desirable level of probability could be determined by reference to structures already standing which are known to have a satisfactory history of serviceability and in which the public has confidence (but which are believed by engineers to be "near the bone"!). The chief merits of the statistical approach are that it is realistic and ensures that similar structures are designed to a consistent standard of safety.

THE RESPONSE OF AN ELASTIC STRUCTURE TO A GUSTY WIND

16. A general expression for the resistance of an object to a flow which is not necessarily steady is:

$$R_t = \tfrac{1}{2}\rho \, . \, C_D \, . \, V_t \, . \, |V_t| + C_m \rho \frac{A_0}{D} \cdot \frac{dV_t}{dt} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where R_t = force per unit area on the object at time t;
V_t = velocity of the fluid at time t;
ρ = fluid density;
D = diameter of the object;
A_0 = a reference area for the virtual mass $\left(\text{generally } \frac{\pi D^2}{4}\right)$;
C_D = coefficient of drag;
C_m = coefficient of virtual mass (including what is known as the additional or associated mass coefficient).

17. The right-hand side of equation (1) contains two terms, the first of which is the familiar expression for the form drag which is proportional to the square of the velocity. The second term is the inertial reaction associated with the acceleration of the fluid. The existence of this latter term has seldom been alluded to in relation to wind loading, although in cases where the wind may change its velocity and direction extremely rapidly as in tornadoes and sharp squalls, it may be at least as important as the drag (Fedyayevsky and Belotserkovsky[12], 1954). These storms, however, represent very special wind-loading problems and will not be considered here. With the more usual levels of gustiness prevalent in high winds the inertial term is probably less important.

18. Suppose further, that the force R_t specified by equation (1) acts upon a simple dynamic system such as that shown in Fig. 1, which possesses one degree of freedom. The equation of motion for such a system is:

$$m\frac{d^2Y}{dt^2} + C\frac{dY}{dt} + k \, . \, Y = R_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where Y_t = deflexion of the system at time t;
m = mass of the system;
C = velocity-damping coefficient;
and k = spring stiffness (load per unit deflexion).

The response (Y_t) of the system to any given input velocity (V_t) is now completely defined by equations (1) and (2). The solutions for two particular types of flow will now be considered.

(a) *Flow with periodic fluctuation*

19. Consider first the simple case in which the flow consists of a mean

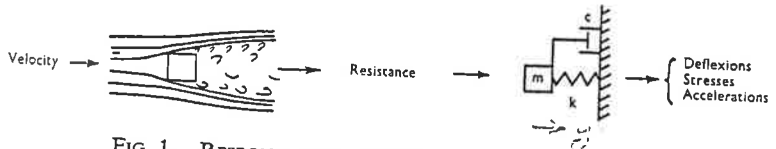


FIG. 1.—RESPONSE OF SIMPLE STRUCTURE TO FLUID FLOW

velocity, $\bar{V}$, with a superimposed sinusoidal fluctuation of amplitude v (small compared to $\bar{V}$) and frequency n. That is:

$$V_t = \bar{V}\left(1+\frac{v}{\bar{V}}\sin 2\pi nt\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

20. Since $v/\bar{V}$ is small so that terms involving v^2 are negligible, equation (1) for R_t is linear in v. Before solving this equation, however, it is necessary to make some comments about the values of C_D and C_m appropriate to this type of flow.

21. It was found in one or two simple experiments by Davenport[13] in 1960 that, although the drag coefficient associated with the mean flow always remains effectively constant at its steady flow, denoted by $C_D(0)$, the coefficients (both of drag and virtual mass) appertaining to the fluctuating part show a dependence on the dimensionless parameter, $\frac{nD}{\bar{V}}$. This is not surprising since the drag is closely linked to the eddy-shedding in the wake of the object which is governed by the same parameter. The parameter is sometimes referred to as the reduced frequency and in the Paper will be denoted by ξ. Typical relations (for an infinite flat plate) between the fluctuation drag and virtual mass coefficients, $C_D(\xi)$ and $C_m(\xi)$, and ξ are shown in Fig. 2.

22. Using this information, the solution to equation (1) can now be written:

$$R_t = \bar{R}\left[1+\chi_a\cdot\frac{2v}{\bar{V}}\cdot\sin\,(2\pi nt+\lambda_a)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $\bar{R} = \frac{1}{2}\rho\,.\,C_D(0)\,.\,\bar{V}^2$

$$\lambda_a = \tan^{-1}\frac{\pi^2 C_m(\xi)\,.\,\xi}{2C_D(\xi)}$$

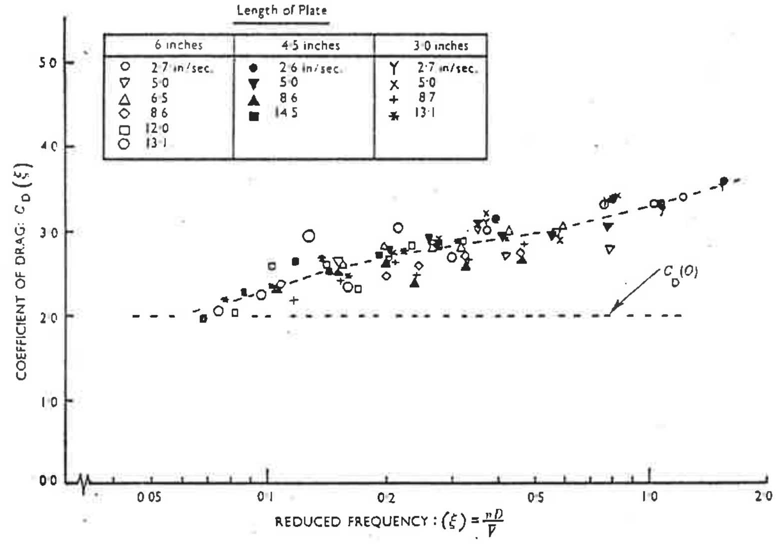


FIG. 2a.—COEFFICIENT OF DRAG OF FLAT PLATE IN FLUCTUATING FLOW

$$\text{and} \quad \chi_a = \frac{\sqrt{C_D{}^2(\xi)+\pi^4/4.\,\xi^2.\,C_m{}^2(\xi)}}{C_D(0)} \quad . \quad . \quad . \quad (5)$$

With this value for R_t, the solution to equation (2) becomes:

$$Y_t = \bar{Y}\left[1+\chi_a\cdot\chi_m\cdot\frac{2v}{\bar{V}}\cdot\sin\left(2\pi nt+\lambda_a+\lambda_m\right)\right] \quad . \quad . \quad . \quad (6)$$

where $\bar{Y} = \bar{R}/k$

$$\lambda_m = \tan^{-1}\left(\frac{\delta}{\pi}\right)\cdot\frac{n/n_0}{1-(n/n_0)^2}$$

n_0 = the natural frequency of the system $= \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

δ = logarithmic damping decrement $= \dfrac{2\pi c n_0}{k}$

$$\text{and} \quad \chi_m = (\{1-(n/n_0)^2\}^2+(\delta/\pi)^2(n/n_0)^2)^{-\frac{1}{2}} \quad . \quad . \quad . \quad (7)$$

(χ_m) is commonly termed the mechanical magnification factor for the system: by analogy (χ_a), will be termed the aerodynamic magnification factor.

(*b*) *Flow with stationary, random fluctuations*

23. In the above example the velocity at any time could be specified explicitly. Another type of flow is now considered in which the velocity is a "stationary, random function". This statement implies that the velocity is random and cannot be predicted from one moment to the next except in so far as it is governed by certain definite laws of probability which are stationary—that is to say, they are independent of the origin of time and of the duration of the record (provided the latter is long enough to prevent sampling errors).

24. This type of flow, it is found, is characteristic of most turbulent flows, and generally of the wind as well. In these circumstances it is useless to try to discuss the response of a structure to a gusty wind in terms of the phase and amplitude of the input and output—as was done in the previous example. Instead it is necessary to determine the statistical properties of the wind velocity and relate to them the deflexions, reactions, and stresses with which the design of the structure is concerned. In general, these statistical properties (e.g. the distribution function, frequency function, etc.) will be defined by certain mean moment values which for the wind velocity are given by:

$$\bar{V}^j = \frac{1}{T}\int_0^T V_t{}^j.dt \quad . \quad . \quad . \quad (8)$$

where T, the duration of the record, is large.

25. Fortunately, it is now fairly well established that velocities in the wind are distributed according to the familiar "normal" or Gaussian law, for which the frequency function is:

$$f(V).dV = \frac{1}{\sqrt{2\pi}}e^{-0\cdot 5x^2}.dx \quad . \quad . \quad . \quad (9)$$

and the distribution function is:

$$F(V) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x_1} e^{-x^2/2}.dx$$

where $$x = \frac{(V-\bar{V})}{\sigma(v)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and $$\sigma(v) = \sqrt{\bar{V}_t^2 - (\bar{V})^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$\sigma(v)$ is known as the standard deviation and $\sigma^2(v)$ as the variance.

26. Evidently, in this case all that is required to define the statistical distribution is a knowledge of the mean velocity, $\bar{V}$, and the mean square velocity, $\bar{V}^2$ (or the variance $\sigma^2(v)$). To find these, it is necessary to refer to the original record of the wind velocity. No matter how irregular this may be, it can always be broken down by Fourier analysis into a series of harmonic sine and cosine terms—or alternatively sine terms involving a phase angle—as follows:

$$V_t = \bar{V}\left[1 + \sum_{j=1}^{\infty} \frac{v_j}{\bar{V}} \sin\left(2\pi j \frac{t}{T} + \theta_j\right)\right]$$

(In this form the expression is now similar to that used in the previous example of periodic flow.)

27. If T, the duration of the record, is increased so that the interval between consecutive frequencies becomes indefinitely small, the summation becomes continuous and may be replaced by an integral. That is:

$$V_t = \bar{V}\left[1 + \frac{1}{\bar{V}} \int_0^{\infty} a(n) \sin \overline{2\pi nt + \theta_n dn}\right] \quad . \quad . \quad . \quad . \quad (12)$$

where n = the frequency

and $a(n)$ = a coefficient having units of velocity per unit frequency.

For this sequence the mean velocity is clearly, $\bar{V}$, and the mean square velocity:

$$\bar{V}_t^2 = \bar{V}^2 + \int_0^{\infty} \frac{a^2(n)}{2} \cdot dn \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

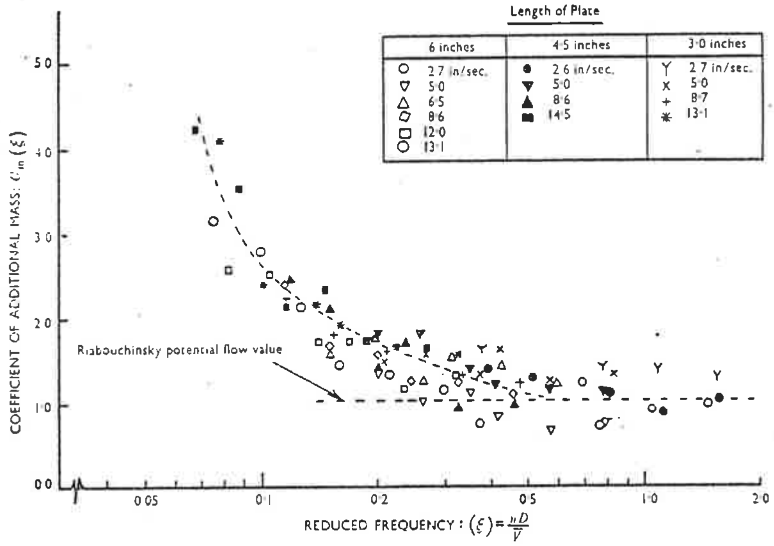


FIG. 2b.—COEFFICIENT OF ADDITIONAL MASS OF FLAT PLATE IN FLUCTUATING FLOW

28. The important quantity $\frac{a^2(n)}{2}$ will be denoted by $S(n)$, and is the spectrum (or power spectral density) of the gustiness and defines the variation of the mean square intensity of the velocity fluctuations with frequency.

29. According to equations (11) and (13) the variance of the fluctuations is now given by:

$$\sigma^2(v) = \int_0^\infty S(n).dn \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

30. In high winds values of $\sigma(v)$ seldom exceed 25% of the mean wind velocity (averaged over an hour or so). It seems therefore reasonable to approximate equation (1) to a linear relation between the resistance and the fluctuating component of velocity as was done in the previous example of a simple periodic fluctuation. According to equation (2) the relation between resistance and deflexion is also linear. Consequently, the distributions of velocity, resistance, and deflexion may be assumed similar. Since the velocity distribution is already known to be Gaussian it follows that the resistance and deflexion are also Gaussian.

31. By direct analogy with the previous results for periodic flow (see equation (4)), the mean resistance and its standard deviation are respectively:

$$\bar{R} = \tfrac{1}{2}\rho . C_D(0) . \bar{V}^2$$

and
$$\sigma(R) = \bar{R}\left[\int_0^\infty |\chi_a(\xi)|^2 . \frac{S(\xi)}{\bar{V}^2} . d\xi\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (15)$$

Similarly the mean deflexion and its standard deviation (cf. equation (5)) are:

$$\bar{Y} = \bar{R}/k$$

and
$$\sigma(Y) = \bar{Y}\left[\int_0^\infty |\chi_a|^2 . |\chi_m|^2 \frac{S(n)}{\bar{V}^2} . dn\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (16)$$

32. Equations (15) and (16) completely specify the statistical distributions of the resistance and the deflexion in terms of the mean wind velocity $\bar{V}$ and the spectrum $S(n)$ (or $S(\xi)$) of the gustiness. The next section is devoted to a discussion of these quantities.

33. It should not be assumed that the aerodynamic magnification χ_a is necessarily the same as that given in equation (5), which referred to a uniformly fluctuating flow in which the velocity varied only with time. If the random fluctuations are turbulent the velocity is also space-dependent, which implies that the fluctuations at different points in the flow differ in phase and in the extent to which their relationship is random. When the size of the body and the flow it disturbs are small compared with the wavelength of the fluctuation, the phase effect is presumably negligible and the effective value of χ_a is the same as that for uniform flow at the same reduced frequency (such as that given by Fig. 2). However, when the wavelength and the object are about the same size, the values of χ_a in uniform and turbulent flow must be expected to differ substantially. The spatial phase effect probably reduces the effective value of χ_a more and more as the disturbances become smaller in relation to the size of the object, whereas in uniform flow at the same reduced frequencies the values of χ_a may become increasingly large—as the trend in the drag coefficient of Fig. 2 suggests. In the illustrative example discussed later an attempt is made to allow for this spatial phase effect in the form chosen for χ_a.

34. From the above it is seen that the determination of the wind loading hinges primarily upon knowing what extreme wind velocity to expect and the form of the spectrum of gustiness. Both these topics have been discussed previously by Davenport[14, 15], and the conclusions then reached are now summarized.

(a) The climate of extreme mean wind speeds

35. It is well known that the wind is induced by the large-scale pressure gradients (shown by the isobars on weather maps) which arise over the earth's surface. The wind moving freely under the influence of these pressure gradients and unaffected by the frictional stresses near the ground surface is known as the gradient wind, and its velocity, the gradient velocity $\bar{V}_G$. The height at which this velocity is first attained (generally between 1,000 and 2,000 ft) is termed the gradient height z_G.

36. Nearer the ground, however, in the region of interest to the structural engineer, the wind is retarded by the surface friction and some of its kinetic energy dissipated in turbulence. It has been suggested (Davenport[15], 1961) that for most occasions of high wind the mean wind speed $\bar{V}_z$ at height z (below z_G) is given by a power law of the type:

$$\bar{V}_z = \bar{V}_G\left(\frac{z}{z_G}\right)^{\alpha} \quad \ldots \ldots \ldots (17)$$

in which z_G and the index α are functions of the ground roughness. Suggested values of these quantities for three types of terrain are given in Table 1. The corresponding profiles (for a uniform gradient wind of 100) are shown in Fig. 3. These show that at 100 ft the mean wind speed in open country may be about twice that in the centre of a city.

TABLE 1.—INFLUENCE OF SURFACE ROUGHNESS ON PARAMETERS RELATING TO WIND STRUCTURE NEAR THE GROUND

Type of surface	Power law exponent α	Gradient height z_G	Drag coefficient K
(a) Open terrain with very few obstacles: e.g. open grass or farmland with few trees, hedgerows, and other barriers, etc.; prairie; tundra; shores, and low islands of inland lakes; desert . .	0·16	900	0·005
(b) Terrain uniformly covered with obstacles 30–50 ft in height: e.g. residential suburbs; small towns; woodland, and scrub. Small fields with bushes, trees, and hedges	0·28	1,300	0·015
(c) Terrain with large and irregular objects: e.g. centres of large cities; very broken country with many windbreaks of tall trees, etc.	0·40	1,700	0·050

37. It seems, then, that although the gradient velocity may change only slightly from place to place (owing to its dependence only on the general circulation patterns of the large-scale pressure systems), the wind speed at the surface may be subject to large local variations. This suggests that in establishing the wind climate of a given area there are advantages in using the gradient velocity as a reference and estimating the velocity at other heights from profiles such as those shown in Fig. 3. Some approach of this kind seems all the more necessary when it is realized that the meteorological stations for which long-term anemometer records are available (on which estimates of extreme wind speed must be based), vary in exposure from the top of a lighthouse in open sea to the centre of a large city. A simple method for establishing the climate (i.e. statistical properties) of the extreme gradient wind velocities, suggested by Davenport[14] in 1960, is now described.

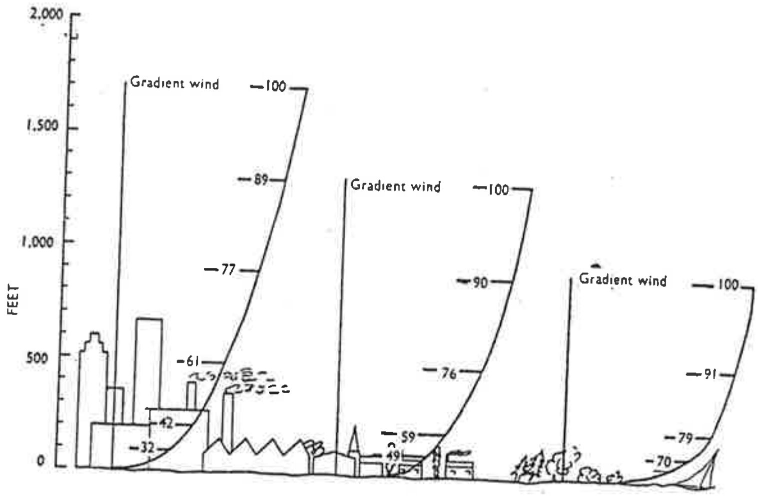


FIG. 3.—VARIATION OF MEAN WIND VELOCITY PROFILE WITH SURFACE ROUGHNESS

38. As a first step, the statistical properties of the surface velocities as given by anemometer records are found. To do this the annual wind-velocity maxima for a number of years are extracted from records of each station and analysed statistically. It is found that their statistical distribution—in common with many other climatological and other extremes—can be satisfactorily represented by the function:

$$P(V) = e^{-e^{-y}} \quad \ldots \ldots \quad (18)$$

where $y = a(V-U)$;

a = the scale factor for the data (measuring its dispersion);

and U = the mode of the data.

Here, $P(V)$ denotes the probability that the maximum velocity in any one year is less than V.

39. Some valuable results of this kind have been compiled by Shellard[16], in 1958, from the anemometer records for selected stations in the British Isles. From his values of the parameters $\frac{1}{a}$ and U for the various stations, estimates of the corresponding parameters for the gradient wind were then made (see Davenport[14], 1960). This was done assuming a constant ratio between the velocity of the gradient wind and that at the anemometer height, which was determined from the wind-velocity profile which seemed most appropriate to the site. Contours of the parameters of the extreme mean hourly gradient wind speed, obtained by these means are shown in Fig. 4. (This is a slightly improved version of the map given previously by Davenport[14] in 1960.)

40. From this map, the gradient-wind velocity with any prescribed probability of occurring can now be predicted. For some uses—such as flood prediction—it has become customary to allude to this probability in terms of a return period which defines the average number of years before a certain value either recurs or is again exceeded. If this convention is adopted and the return period is denoted by r, the probability that a certain value is *not* exceeded in any one year is $\left(1-\frac{1}{r}\right)$. Hence from equation (18) the gradient-wind velocity $\bar{V}_G$ for which r is the return period is:

$$\bar{V}_G = U-\frac{1}{a}\left[\log_e\left(-\log_e \overline{1-\frac{1}{r}}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad (19)$$

For large values of $r(r>10)$ this can be approximated by:

$$\bar{V}_G = U+\frac{1}{a}\cdot\log_e r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The values of U and $\frac{1}{a}$ are read from the contours of the map.

41. The extreme mean wind velocity closer to the ground may now be determined from the power law:

$$\bar{V}_z = \bar{V}_G\left(\frac{z}{z_G}\right)^{\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where, of course, the values of z_G and α are selected from Table 1 according to the roughness of the site.

42. In the next part the complementary problem of predicting what gustiness will occur simultaneously with this mean hourly wind velocity is considered.

(b) The spectrum of gustiness

43. An expression proposed by Davenport[15] in 1961 for the spectrum of horizontal gustiness in high winds at height z is:

$$S_z(n).dn = 4{\cdot}0\,K.\bar{V}_1{}^2\cdot\frac{x}{(1+x^2)^{4/3}}\cdot dx \quad . \quad . \quad . \quad . \quad (22)$$

where $x = 4{,}000\, n/\bar{V}_1 \left(\frac{n}{\bar{V}_1} \text{ in cycles/ft}\right)$

z_1 = 10 m (33 ft) the standard reference height;

$\bar{V}_1$ = the mean hourly wind velocity at height z_1;

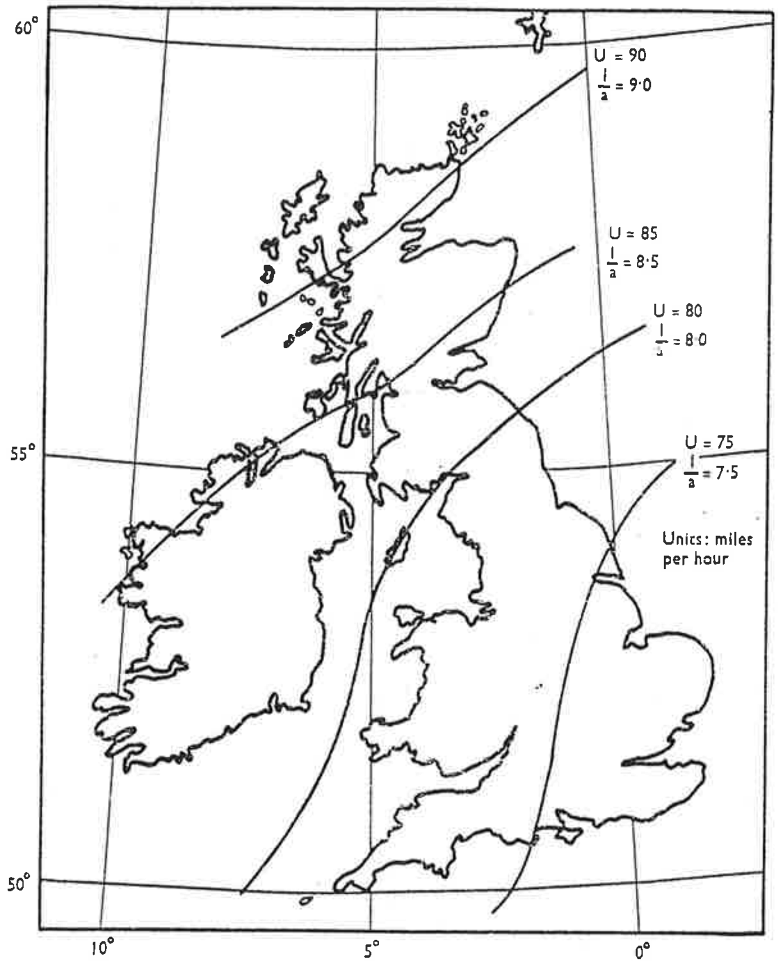


FIG. 4.—PARAMETERS OF EXTREME MEAN-HOURLY GRADIENT WIND SPEED OVER THE BRITISH ISLES

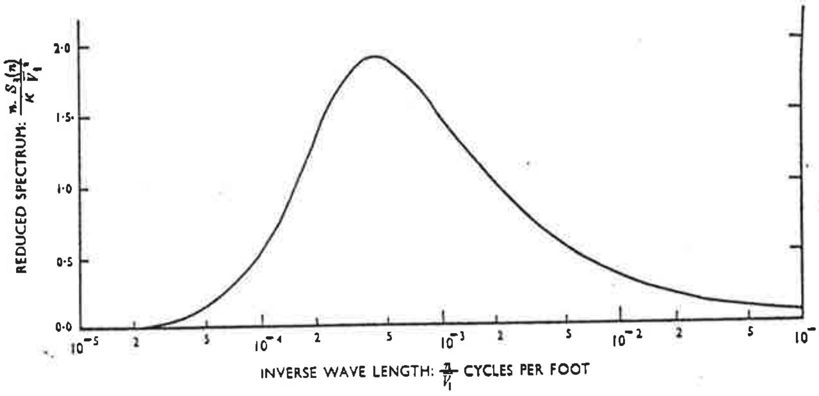
K = the drag coefficient for the surface (referred to the mean velocity at height z_1).

The only new quantity introduced by this expression is the drag coefficient, K, for the surface. Values of this for the three typical grades of surface roughness already referred to are given in Table 1.

44. The general shape of the spectrum can be seen from Fig. 5, only in this instance an alternative representation is used—the product of the spectral density and the frequency. The advantage of this is that when represented on a logarithmic frequency scale—as is appropriate in the present case—the area under the curve still gives a true measure of the energy. This can be seen from the equality:

$$\int n.S(n)d.\log(n) = \int S(n).dn$$

(This fact is particularly useful in planimetric integration.)



Note:
$S_z(n)$ = Spectrum at height z and frequency n
$\bar{V}_1$ = Mean velocity at reference height z_1 = 10m (33 ft)
K = Surface drag coefficient (referred to $\bar{V}_1$)

FIG. 5.—SPECTRUM OF HORIZONTAL GUSTINESS

45. The following are particular features to note about the spectrum:—

(i) Almost all the energy is confined to wavelengths less than 5,000 or 10,000 ft. This implies that in a wind of about 60 m.p.h. the fluctuations with periods less than one or two minutes are small and contribute little to the total energy.

(ii) The spectrum is proportional to the quantity $K\bar{V}_1^2$, which itself is proportional to the shear stress between the air and the ground. This implies that the turbulence is predominantly mechanical rather than convective in origin, which for high winds appears to be substantially true.

(iii) The inverse wavelength (i.e. wave number $n/\bar{V}_1$) is used on the assumption that the spatial pattern of the turbulence remains invariant with change in mean wind velocity. This is true, for example, of flow behind grids, etc.

46. In the equations derived for the wind loading, the spectrum always appears divided by the square of the mean wind velocity. In this form the spectrum can be written:

$$\frac{S_z(n)}{\bar{V}_z^2}\cdot dn = 4{\cdot}0\, K\left(\frac{z}{z_1}\right)^{-2\alpha} \frac{x}{(1+x^2)^{4/3}}\cdot dx \quad . \quad . \quad . \quad . \quad (23)$$

where $x=4{,}000\dfrac{n}{\bar{V}_1}$, and $z_1=33$ ft (10 m) as before. The square root of the integral of this expression defines what is known as the intensity of turbulence denoted $I_z(v)$; that is:

$$I_z(v) = \frac{\sigma_z(v)}{\bar{V}} = \left[\int_0^\infty \frac{S_z(n)}{\bar{V}_z^2}\, dn\right]^{\frac{1}{2}}$$

$$= 2{\cdot}45\, K^{\frac{1}{2}}\left(\frac{z}{z_1}\right)^{-\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

This shows that the intensity of turbulence and the normalized spectrum of turbulence are both independent of wind velocity and dependent only on the height and the parameters of the roughness of the terrain. This helps to simplify the problem.

Time-history relationships

47. The results of this investigation into the response of a simple structure to a gusty wind have so far been, first, to derive expressions for the deflexions and forces in terms of the mean wind velocity and the gust spectrum, and secondly, to provide the necessary data for estimating the latter quantities. It is now shown how this information can be used to answer various questions of practical importance such as: What maximum stresses, deflexions, and accelerations are likely to occur in the structure? How frequently is a given value of stress likely to be exceeded? Several mathematical expressions are now given which assist in answering these questions.

(a) The number of excesses per unit time

48. Rice[17] (1945) has shown that, for a stationary random series of the normally distributed stochastic variable x, the number, $N(x_0)$, of times per unit time, a given value x_0 is exceeded is:

$$N(x_0) = \frac{\sigma'(x)}{\sigma(x)}\, e^{-x_0^2/2\sigma^2(x)} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where $\sigma(x)$ = standard deviation of x

$$= \left[\int_0^\infty S_x(n).dn\right]^{\frac{1}{2}}$$

$$\sigma'(x) = \left[\int_0^\infty n^2.S_x(n)dn\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and $S_x(n)$ = spectrum of x.

(b) The distribution of the peak values occurring within a given period

49. If a number of periods each of duration T are chosen from the same stationary random series, the proportion of them in which the largest values are

less than x_0 is $1-TN(x_0)\left(0 \leqslant N(x_0) \leqslant \frac{1}{T}\right)$. Using Rice's expression for $N(x_0)$ above, it follows that the distribution of the peak, or largest, instantaneous values η for all the periods is:

$$Q(\eta) = 1-\nu T e^{-\eta^2/2\sigma^2(x)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

where $Q(\eta)$ = the probability that the maximum value during period T is less than η.

$\sigma(x)$ = the standard deviation of the parent population of variable x.

and $\nu = \dfrac{\sigma'(x)}{\sigma(x)}$

The frequency density function is:

$$q(\eta) = \frac{\nu T}{\sigma(x)} \cdot \eta . e^{-\eta^2/2\sigma^2(x)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

ν will be termed the response factor. The mean of this distribution is found to be:

$$\bar{\eta} \approx \sigma(x)\left[\sqrt{2 \log_e \nu T}+\frac{1}{\sqrt{2 \log_e \nu T}} \ldots\right] \quad . \quad . \quad . \quad (29)$$

and the standard deviation:

$$\sigma(\eta) \approx \frac{\sigma(x)}{\sqrt{2 \log_e \nu T}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

The form of the distribution and its relation to the parent population can be seen from Fig. 6.

(*c*) *The distribution of largest average values occurring within a given period*

50. If, instead of the peak values, the largest value averaged over a finite interval of time, ΔT, is required the same formulae as above in section (*b*) are used, only a modified form of spectrum for determining $\sigma(x)$ and $\sigma'(x)$ is adopted. This is:

$$S_x'(n) = S_x(n)\left[\frac{\sin n.\pi.\Delta T}{n.\pi.\Delta T}\right]^2 \quad . \quad . \quad . \quad . \quad . \quad (31)$$

APPLICATIONS

51. The expression that has been derived for the probability distribution of the peak values (equation (27)) is seen to be a function of the simultaneous mean value, which, in the present case, is itself a chance occurrence with a probability distribution of its own. Therefore to find the absolute probability of a given maximum requires knowledge of the combined probability of the mean values and their associated maxima. This is not impossible to determine knowing the properties of each distribution separately. In practice, however, the labour involved in the calculations is hardly justified, and it is felt that in most cases perfectly adequate estimates can be made using certain approximations which enable the complications to be avoided.

52. As can be seen from Fig. 6, the distribution of maximum values is a narrow one, and in practical cases 95% of the peak values lie within an interval much less than half the standard deviation of the parent population on either side of the mean peak value. This suggests that not much error will be involved if this slight spread of peak values is disregarded and the peak value in any given period is taken equal to the mean peak value. The total peak value X_p (referred to the true origin) can then be written:

$$X_p = \bar{x}\left[1+g(\nu T)\cdot\frac{\sigma(x)}{\bar{x}}\right]$$

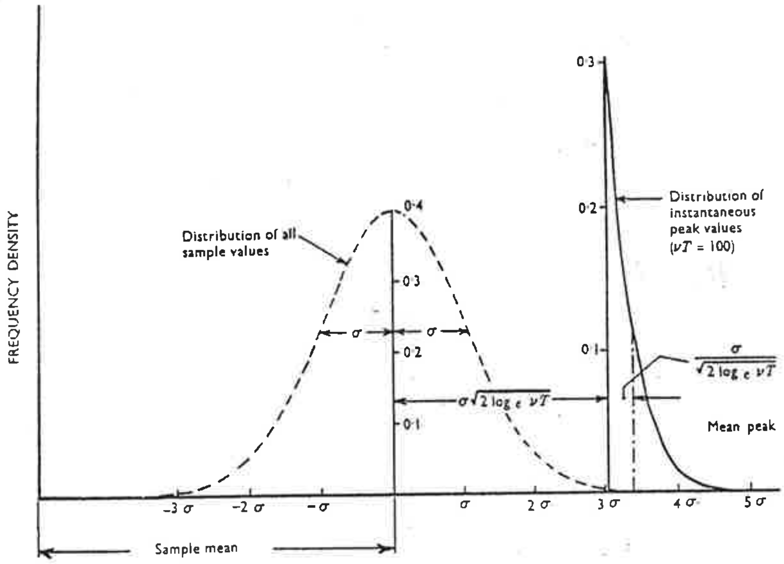


FIG. 6.—RELATIONSHIP BETWEEN THE FREQUENCY DISTRIBUTIONS OF ALL SAMPLE VALUES AND OF MAXIMUM INSTANTANEOUS (PEAK) VALUES FOR ν T (i.e. RESPONSE FACTOR AND SAMPLE DEVIATION) = 100

where $\bar{x}$ = mean value of population and (from equation (29) and Fig. 6):

$$g(\nu T) \approx \sqrt{2 \log_e \nu T} + \frac{1}{\sqrt{2 \log_e \nu T}} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

In general, $g\cdot\frac{\sigma(\bar{x})}{\bar{x}}$ is not necessarily constant, and may have a slightly different value for every value of $\bar{x}$. However, in practical cases the variation is not likely to be great and it will normally be adequate to choose a suitably conservative value corresponding to a value of $\bar{x}$ close to the mean value of the $\bar{x}$ distribution. The peak values will then have a distribution exactly similar to the mean values only the variable will be increased by the factor $\left(1+g\cdot\frac{\sigma(x)}{\bar{x}}\right)$.

53. It can be shown that in practical cases the assumptions this involves are not severe. Measurements of peak gust velocities, for example, appear to follow the extreme-value distribution (equation (18)) just as well as the mean velocities themselves, and the two velocities are in more or less a constant ratio to one another for all return periods.

54. The quantity $\left(1+g\cdot\frac{\sigma(x)}{\bar{x}}\right)$ will be defined as the gust factor since it measures the additional deflexion, force, or velocity which can be attributed to the gustiness of the wind. It is a crucial quantity in estimating wind loads. Its determination is now illustrated for two structures having similar aerodynamic properties but of contrastingly different flexibilities.

Comparison of wind loads on an arc lamp standard and a water tower under different conditions

55. The two structures which will be considered are shown in Fig. 7. The arc-lamp is typical of those found at football stadia, railway marshalling yards, etc. Constancy of illumination may be one of the requirements and the deflexions which may arise in strong or even moderate winds may therefore be almost as important a consideration as the maximum stresses. Since their construction is generally flexible, and the lamps relatively heavy, a low natural frequency

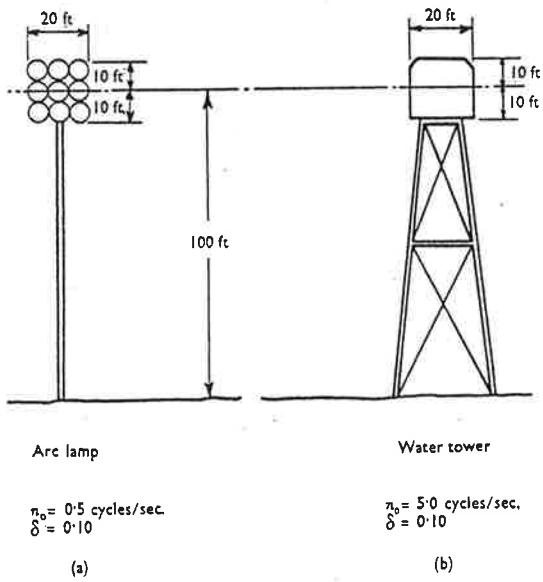


FIG. 7.—DETAILS OF STRUCTURES SUBJECTED TO WIND LOADS

(0·5 c/sec) has been chosen. A water tower on the other hand is generally a fairly rigid type of structure and a much higher natural frequency (5·0 c/sec) has therefore been suggested. The logarithmic decrement of 0·10 is probably typical of civil engineering structures oscillating in high winds. A substantial part of this damping is aerodynamic in origin and can be estimated[13].

56. The aerodynamic resistance has, in each case, been assumed to act at the centre of the superstructure. The aerodynamic magnification $|\chi_a|^2$ and its variation with the reduced frequency ξ, has largely been inferred from the results shown in Fig. 2 for a flat plate. A certain amount of guess work had to be used in adapting these results, applicable to uniform velocity fluctuations, to the case in which the fluctuations were turbulent*. (This distinction has been discussed already in § 33.) From Fig. 2, it is seen that for reduced frequencies less than 0·1 the drag coefficient $C_D(\xi)$ attains the quasi-steady value. Since this corresponds to wavelengths many times the diameter of the structure, the same result is likely to be true of turbulent flow. For reduced frequencies greater than 0·1, Fig. 2 indicates that the drag coefficient slowly increases. However, in turbulent flow this tendency is counteracted by the fact that as the size of the eddies (i.e. their wavelengths) becomes smaller compared to the size of the object their effect eventually must become negligible. To allow for this, the basic form of $C_D(\xi)$ shown in Fig. 2 has been modified using an averaging function similar to that in equation (31) so that in the region $\xi > 1$ it has practically no value. Virtual mass effects have been omitted since these are likely to be small. The resulting form of $|\chi_a|^2$ (assumed the same for both structures) is shown in Fig. 8e. Although it can only be considered as tentative (owing to lack of experimental information) it is believed that the errors involved are not likely to seriously influence the overall estimate of the wind loading.

57. Fig. 8a represents the gust spectrum for the wind velocity input. The resistance spectrum shown in Fig. 8c is obtained directly from the wind-velocity spectrum using $|\chi_a|^2$ as a multiplier. The area (measured planimetrically) is proportional to the required variance of the resistance.

58. The mechanical magnification which has been determined from equation (7) is shown in Fig. 8d, for both structures. Since the ξ-value at which the resonant peak occurs, is fixed by the mean velocity, some flexibility in the final stage of the analysis is introduced. (It was noted earlier that it is probably not worthwhile to investigate all the values of the gust factor for all possible mean wind velocities.) Instead the mean wind velocity is chosen (lying within the range of expected extreme wind speeds) which leads to the largest amplification. This can be done by inspection and normally entails lining up any peaks of the resistance spectrum and the mechanical magnification. In the present case 100 ft/sec seemed a reasonable choice to make for both structures.

59. The resulting spectrum of deflexion is then given by Fig. 8e for both structures. It is seen that the deflexion of the arc-lamp has been amplified considerably, but negligibly so in the case of the water tower. The intensity of the deflexion fluctuations are as follows:

Arc lamp: $$I(Y) = \frac{\sigma(Y)}{\bar{Y}} = 4{\cdot}88\, I_z(v)$$

Water tower: $$I(Y) = 2{\cdot}05\, I_z(v)$$

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* This suggests the need for research into the fluctuating drag in turbulent flow.

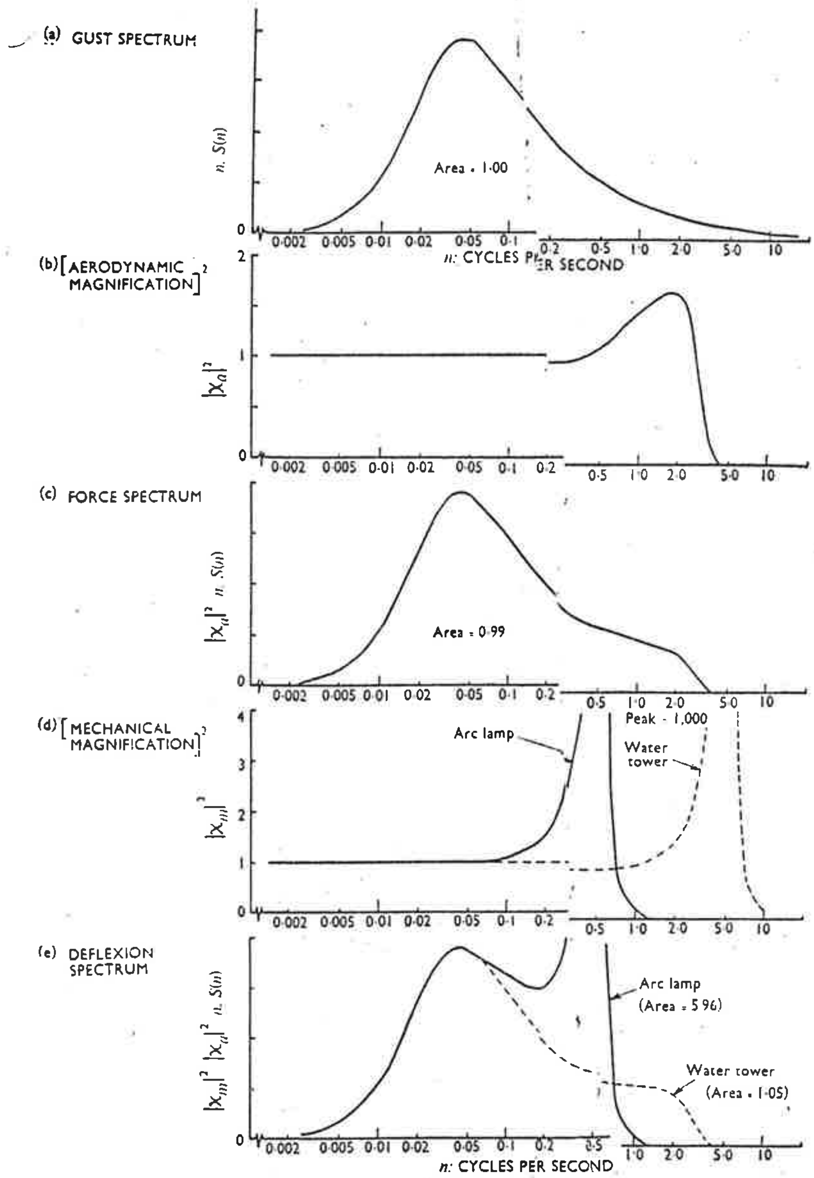

FIG. 8.—RESPONSE OF ARC LAMP AND WATER TOWER TO GUSTY WIND (MEAN SPEED = 100 FT/SEC)

60. The next step is to determine the response factor which requires evaluating:

$$\sigma'(Y) = \left[\int_0^\infty n^2 . |\chi_a|^2 . |\chi_m|^2 . S(n) . dn .\right]^{\frac{1}{2}}$$

61. These spectra are soon integrated planimetrically and the required values are then as follows:

Arc lamp: $\frac{\sigma'(Y)}{\bar{Y}} = 3{\cdot}60\, I_z(v)$

Water tower: $\frac{\sigma'(Y)}{\bar{Y}} = 1{\cdot}73\, I_z(v)$

Accordingly the response factors are:

Arc lamp: $\nu = \frac{\sigma'(Y)}{\sigma(Y)} = 0{\cdot}737$

Water tower: $\nu = 0{\cdot}845.$

62. The values of g in the gust factor $\left(1+g\cdot\frac{\sigma(Y)}{\bar{Y}}\right)$ for a 1-hour period (T=3,600 sec) are (from equation (32)):

Arc lamp: $g = \sqrt{2 \log_e \nu T} + \frac{1}{\sqrt{2 \log_e \nu T}} = 4{\cdot}22$

Water tower: $g = 4{\cdot}25$

63. The gust intensity at the height of the structure (100 ft) is now calculated from equation (24) using the parameters in Table 1 for city and open-country conditions:

Open country: $I_z(v) = 2{\cdot}45\, K^{\frac{1}{2}}\left(\frac{z}{z_1}\right)^{-\alpha} = 0{\cdot}145$

City: $I_z(v) = 0{\cdot}351$

64. The gust factors $\left(1+g\cdot\frac{\sigma(Y)}{\bar{Y}}\right)$ are now found to be:

	Open country	*City*
Arc lamp	3·99	8·23
Water tower	2·26	4·31

65. Suppose now these structures are to be erected at a locality situated on the $\left(U=80; \frac{1}{a}=8{\cdot}0\right)$ contour shown in Fig. 4, and are required to resist the once-in-50-years wind load. With r=50 in equation (20) the required mean hourly gradient wind for this region is:

$$V_G = 80 + 8{\cdot}0 \log_e 50$$
$$= 111{\cdot}5 \text{ m.p.h.}$$