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NATURAL CONVECTION THROUGH RECTANGULAR OPENINGS IN PARTITIONS—1

VERTICAL PARTITIONS

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Abstract—The simple basic theory of natural convection across openings in vertical partitions has been generalized to include both heat and mass transfer. Experiments were carried out in a large heat-flow apparatus with openings from 3 to 12 in high and with air as the convecting fluid. Because of the high thermal resistance of the partition, the test results for the Nusselt number as a function of the Grashof number based on opening height in the range $10^6 < Gr < 10^8$ would also be expected to be directly applicable to mass transfer.

NOMENCLATURE

A, B, C, dimensionless constants;
a, b, exponent;
c_p, specific heat of fluid;
c, concentration; c_1, c_2 concentrations in cavities 1 and 2 remote from the partition;
d, exponent;
D, diffusion coefficient of mass transfer;
E, opening area;
g, acceleration due to gravity;
H, opening height (vertical partition); thickness of horizontal partition;
h_T, heat-transfer coefficient based on total opening area;
h_m, mass-transfer coefficient based on total opening area;
k, thermal conductivity of fluid;
l, pressure loss;
L, width of opening in horizontal partition;
$\dot{m}$, mass-transfer rate through opening;
$\dot{q}$, heat-transfer rate through opening;
p, pressure;
Q, volumetric fluid-flow rate;
S, vertical distance between multiple openings;
T, temperature; T_1, T_2 temperatures in cavities 1 and 2 remote from the partition;
t, thickness of vertical partition;
V, velocity;
W, width of opening in vertical partition;
z, distance in the direction of gravity;
Z, $= \Delta\rho z/\bar{\rho}$ = height of a fluid column of density $\bar{\rho}$ giving a pressure under gravity equal to $\Delta\rho g z$;
α, thermal diffusivity;
β_m, β_T, coefficient of concentration and thermal expansion respectively;
ϵ, ξ, exponents;
μ, dynamic viscosity;
ν, kinematic viscosity;
ρ, fluid density; ρ_1, ρ_2, density in cavities 1 and 2 remote from the partition; $\rho_1 - \rho_2 = \Delta\rho$, difference in density between the fluid in the two cavities; $\bar{\rho} = (\rho_1 - \rho_2)/2$, mean density of fluid.

Dimensionless groups

Gr, Grashof number based on density difference, $g\Delta\rho H^3/\bar{\rho}\nu^2$;
Nu, Nusselt number, $h_T H/k$;
Pr, Prandtl number, $c_p\mu/k$;
Re, Reynolds number, VH/ν;
Sc, Schmidt number, ν/D;
Sh, Sherwood number, $h_m H/D$.

Similarly the pressure difference at a level z above the centerline is just the negative of equation (3). The pressure difference $p_1 - p_2$ can also be expressed as the height of a column of fluid

$$Z = \frac{(\rho_1 - \rho_2)z}{\bar{\rho}} = \frac{\Delta\rho}{\bar{\rho}} z$$

where $\bar{\rho}$ is the mean density

$$\left(= \frac{\rho_1 + \rho_2}{2}\right). \tag{4}$$

There is only limited information available for the relation between pressure head and velocity V for rectangular orifices at low flow rates. Consequently the flow will first be assumed to be ideal (frictionless), and then the influence of viscous forces will be considered.

For ideal flow the Bernoulli equation can be assumed, i.e.

$$V = \sqrt{(2gZ)} = \sqrt{\left(2g\frac{\Delta\rho}{\bar{\rho}}z\right)}. \tag{5}$$

On integration from $z = 0$ to $z = H/2$, equation (5) gives for the total volumetric discharge through one half of the opening

$$Q = C\frac{W}{3}\sqrt{\left(g\frac{\Delta\rho}{\bar{\rho}}\right)}H^{3/2}. \tag{6}$$

The coefficient of discharge C has been inserted here as is customary for orifices. The value of C ranges from about 0·6 for sharp-edged orifices to 0·8 for short tubes, and up to 0·98 for trumpet shapes. For submerged orifices, C tends to take on higher values.

With the flow Q is now associated:

the heat-transfer rate

$$\dot{q} = Q\bar{\rho}c_p(T_1 - T_2), \tag{7}$$

the mass-transfer rate

$$\dot{m} = Q\bar{\rho}(c_1 - c_2), \tag{8}$$

where c_p is the specific heat.

Introducing now the heat-transfer coefficient h_T and the mass-transfer coefficient h_m, defined as

$$h_T = \dot{q}/WH(T_1 - T_2)$$

and

$$h_m = \dot{m}/WH\bar{\rho}(c_1 - c_2),$$

equations (7) and (8) lead to the following equations in terms of dimensionless variables:

for heat transfer

$$Nu = \frac{h_T H}{k} = \frac{C}{3}\sqrt{\left(\frac{g\Delta\rho H^3}{\nu^2\bar{\rho}}\right)}\cdot\frac{c_p\mu}{k}$$

$$= \frac{C}{3}\cdot\sqrt{(Gr)}\,Pr, \tag{9}$$

for mass transfer

$$Sh = \frac{h_m H}{D} = \frac{C}{3}\sqrt{\left(\frac{g\Delta\rho H^3}{\nu^2\bar{\rho}}\right)}\frac{\mu}{\bar{\rho}D} = \frac{C}{3}\cdot\sqrt{(Gr)}Sc \tag{10}$$

where the symbols are as defined in the Nomenclature.

Equations (9) and (10) cannot be exact for all conditions owing to neglect of viscosity in equation (5) and neglect of thermal conductivity and diffusivity in equations (7) and (8). Considering first viscosity,* it will be remembered that for pure viscous flow velocity is directly proportional to the difference in pressure head, just as for laminar flow in a tube or viscous flow about a submerged object a limiting equation will occur of the form

$$Q = AW\frac{\Delta\rho}{\bar{\rho}}g\frac{H^3}{\nu}, \tag{11}$$†

where A is a dimensionless constant.

Using equation (11) in place of equation (5) the Nusselt and Sherwood numbers become

$$Nu = AGrPr,$$

$$Sh = AGrSc. \tag{12}$$

If now the usual procedure is adopted and the Nusselt number is written

$$Nu = BGr^a Pr^b$$

for a small range of the variables, then by equations (9) and (12) for negligible thermal

* The following considerations are similar to those given in [4] for heat exchange in a vertical tube at low flow rates.

† A similar equation showing Q directly proportional to $(\Delta\rho/\bar{\rho})g(H^3/\nu)$ can be derived from first principles for the case of a vertical slot.

conductance the exponent a on the Grashof number lies between $\frac{1}{2}$ and 1 and the exponent b on the Prandtl number is practically 1. The same reasoning holds for mass transfer.

For the other extreme, wherein the thermal conductivity or diffusion coefficient is very large, the heat- or mass-transfer phenomenon is similar to that of a solid, i.e.

$$Nu = \text{const.} \quad Sh = \text{const.} \tag{13}$$

Consequently, for the complete range of all variables (all fluids), the exponents on both Gr and Pr in equation (13) lie between 0 and 1. To be somewhat more specific about these exponents for various fluids some additional information can be obtained by comparing the estimated pure conduction or diffusion exchange obtained by analytical means with the convective transfer of equations (7) and (8). For air, for example, in the range of greatest interest, with say 50 degF temperature difference across a 1-ft square opening, the pure conduction heat transfer would be quite negligible compared with that of convection. Furthermore, the value of the Reynolds number Re, based on the mean flow velocity as determined from equation (6), lies close to the range covered in experiments on orifices which themselves validate equations (5) and (6). Hence, for air in this general range the final form of equation (13) is expected to be approximately

$$Nu = (0{\cdot}6 \text{ to } 1{\cdot}0)\tfrac{1}{3}Gr^{\frac{1}{2}-\epsilon}Pr^{1-\xi} \tag{14}$$

where ϵ and ξ are small compared with $\frac{1}{2}$ and 1, respectively.

Multiple openings

Multiple openings in the vertical direction will increase the mass or heat transfer. In general, determination of the convection becomes complicated when openings of various sizes at irregular spacing are present. For equally spaced openings of the same size, however, it will be noted that the level of equal pressure in the two cavities falls on the center of gravity of the openings. Multiple openings will be discussed in greater detail in the following section on experimental results.

EXPERIMENTAL

Equipment

The test unit, already described in detail [3], consists of two large chambers 8 ft square, one of which is 4 ft deep and the other, since it contains an additional system to provide forced air circulation at will, 14 in deep. The first of these chambers can be maintained at approximately 70°F and the other at variable temperature down to −20°F. The walls of the unit are insulated and guarded to prevent heat losses. Fig. 2 shows the complete apparatus and its control equipment.

The partition in which the openings were cut was constructed of 2-in foamed polystyrene insulation on a $\frac{1}{4}$-in plywood backing and was clamped and sealed between the two boxes of the test unit. Use of this wall of high thermal resistance has two experimental advantages: (1) the heat transfer across the opening will be a large fraction of the total heat transfer, and (2) the large natural convection in the neighborhood of the opening will not be greatly influenced by convection over the wall proper. The convection conditions, then, approximate closely those which would occur when density differences are due to concentration differences alone, and the test results for heat transfer would be expected to apply for mass transfer as well.

Instrumentation

Thirty-gauge copper–constantan thermocouples were installed at various locations to measure air and surface temperatures on both sides of the partition. The temperature control was sufficient to maintain the air temperatures remote from the openings constant with time to within 0·2 degF. Total heat input to the warm side of the test unit was determined from continuous recordings of d.c. voltage and current, the accuracy of the power input thus obtained being about 2 per cent.

Scope of tests and procedure

Tests were conducted for single rectangular openings of the following nominal sizes: 6 × 6 in, 6 × 12 in, 9 × 9 in and 12 × 12 in, with air temperature differences ranging from 15 to 85 degF. (Temperature on the warm side was maintained throughout at about 72°F.) Tests were also made for 3 × 3-in openings but, since the heat flow for a single opening of this size would be very small, seven openings were spaced horizontally on $10\frac{1}{2}$-in centers. The ratio t/H of partition thickness t to opening height H would be expected to affect the magnitude of results, and was varied by altering the partition thickness in the neighborhood of the opening, either by removing or adding insulation. Account was taken of the changed heat transmission through this portion of the wall. Several tests were also made with two equal 3 × 3-in openings spaced vertically 15 in apart on centers.

In practical installations some forced air movement in addition to natural convection might be present. Since the test unit already contained a closed forced-air circulation system on the cold side, it was used to obtain additional test results with air flowing horizontally parallel to the partition and opening with velocities of 100 and 200 ft/min.

Owing to the massiveness of the test apparatus, long periods of time were required to reach equilibrium conditions. In general, with natural convection alone the apparatus was allowed to run overnight before temperature and heat-flow readings were taken. On completing a natural convection test at a given temperature difference, forced air circulation was begun and equilibrium was again reached in a period of 3 or 4 h.

Before carrying out tests with the various openings it was first necessary to calibrate the blank partition. This was done by determining the total heat transfer at various temperature differences between the air on the two sides of the wall. Results are shown graphically in Fig. 3, where the total heat flow has been divided by the partition area (64 ft²) to obtain conductance. The slight irregularity in the test results with no forced air motion is due to the unavoidable variability of the film coefficients. This assertion is borne out by the tests with air velocities of 100 and 200 ft/min which show no irregular behaviour.

With an opening in the partition, a small portion of the total heat transfer occurs by radiation, the amount of which was calculated

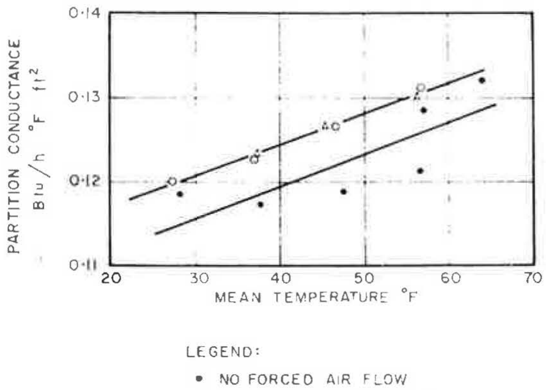

FIG. 3. Calibration of the test partition.

by assuming that both cavities behave as black bodies, i.e.

$$\dot{q}_r = WH\,\sigma(T_{1_s}^4 - T_{2_s}^4) \qquad (15)$$

where σ = the Stefan–Boltzmann constant and subscripts denote cavity surface conditions. (Interchange with the edges of the opening was neglected.)

Test results

Natural convection (no forced-air flow). The air temperatures used in determining the Nusselt numbers were the averages from floor to ceiling on the two sides of the wall. Air properties were evaluated at the mean air temperature of both sides. Air temperatures were measured 15 in from the wall on the warm side and 8 in from the wall on the cold side. The temperature-measurement stations were 2 ft from the opening in a direction parallel to the wall. In the case of the 12 × 12-in opening the heat-transfer rate was sufficiently large to cause a considerable gradient in the remote air temperature from floor to ceiling on both sides of the partition (Fig. 4). Since the two gradients did not differ greatly, however, it will be appreciated that only a small error is involved in basing all calculations on mean temperatures. Fig. 4 also shows the distortion of the air and surface temperatures in the plane of the opening center line caused by the double air flows.

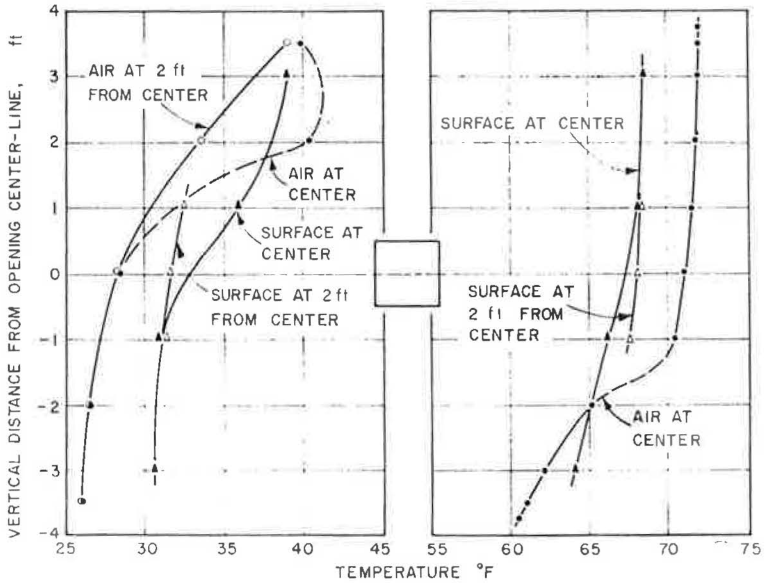

FIG. 4. Air and surface temperatures about a 12 × 12-in opening in a $2\frac{1}{4}$-in insulated partition.

All experimental results are given in Fig. 5, where the Nusselt number divided by the Prandtl number is ordinate and the Grashof number is abscissa in accordance with equations (9), (12), (13) and (14). For comparison, the two theoretical extremes of equation (9) with $C = 0{\cdot}6$ and $1{\cdot}0$ are shown. In Fig. 5, the influence of the additional variable, the ratio t/H of the wall thickness t to the opening height H, is apparent. It will be noted that for any given t/H ratio the slope of a line through the experimental results is always greater than 0·5 in accordance with equation (14). Furthermore, the slope increases with decreasing Grashof number, i.e. tending more and more toward a value of 1·0 for very low values of Gr. At the higher values of the Grashof number (between 10^7 and 10^8) very little influence of t/H is to be noted for the range $t/H = 0{\cdot}19$–$0{\cdot}38$. This is further borne out by the test results for the rectangular opening 6 in high and 12 in wide; they are apparently identical with those for the 6 × 6-in square. For $Gr < 10^7$ and for a greater range of t/H from 0·38 to 0·75, the influence of t/H is marked.

Since the Prandtl number for air in the range of temperatures used in the tests was constant at 0·71 it was not possible to investigate its influence as a separate variable. A comparison of the Nusselt numbers of the present work with those of natural convection over a vertical plate for the same range of Grashof numbers, however, shows very much higher values for the case of wall openings. For the vertical plate the thermal conductivity of the fluid plays a large role in the heat exchange, in that heat is transferred by pure conduction in a laminar layer of fluid on the plate, and it is found that the Nusselt number is proportional to $Pr^{0{\cdot}25}$ or $Pr^{0{\cdot}33}$. With the much higher values of Nu in the present work indicating a greater convection transfer, it would be expected that for most liquids and gases the exponent on Pr (or Sc) is close to 1·0 in accord with equations (9) and (12). (For a situation where turbulent flow occurs and in which the exponent on Pr is even less than 0·25, see [4] for the case of heat transfer in a vertical tube.)

A series of tests was carried out with two 3 × 3-in openings spaced vertically 15 in between centers in the $2\frac{1}{4}$-in partition. In this case, the Nusselt number based on the total

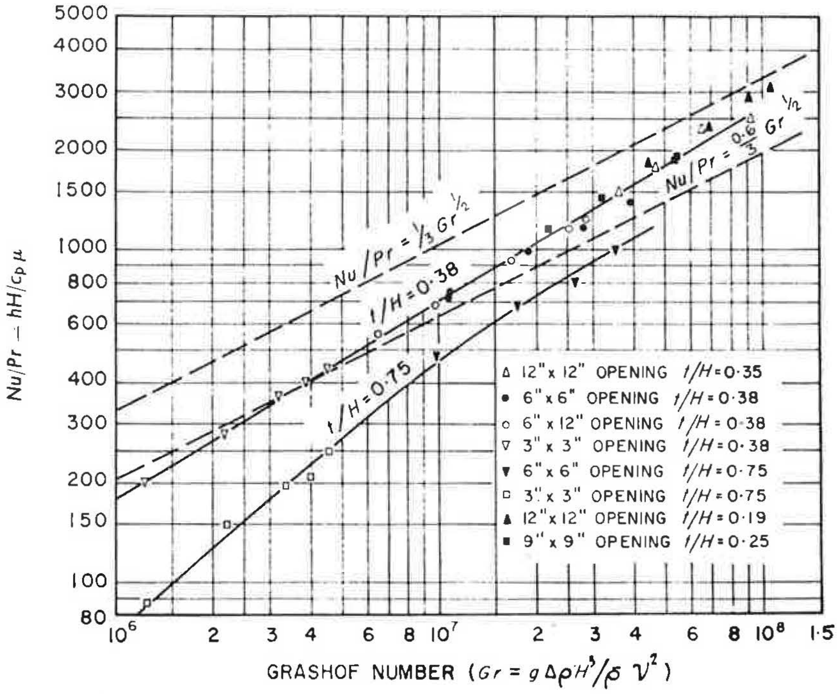

FIG. 5. Natural convection across rectangular openings in a vertical partition.

area of the two openings and $H = 3$ in should be equal to one-half the difference between the Nusselt numbers for two single openings of height 18 and 12 in (see Appendix A). In Fig. 6 the measured values are plotted against theoretical values as obtained by extrapolation from Fig. 5. Agreement is fairly good, but the measured values are somewhat lower than theoretical ones, presumably owing to the additional contracting effect of the narrow 3-in width. Since the ratio t/H used in these tests had the high value of 0·75 it can be expected that agreement between theory and measurement would be appreciably better for lower (more usual) t/H values.

Effect of a horizontal velocity parallel to the wall. It was not the intention in this work to study the effects of forced-air motion in detail but only to make approximate measurements. This restriction was demanded in part by the test apparatus itself, which allowed use of only one or two fans, each of 1000-ft^3/min capacity. The cross-sectional area for flow on the cold side of the wall being 10 ft^2, average horizontal velocities of 100 and 200 ft/min were available for the tests. Another consideration restricting the quantitative applicability of the test results is the geometry of the cold side box and the roughness of the wall; together these determine the velocity of approach profile at the wall opening.

The test results are given in Fig. 7, along with the superimposed mean curve for natural convection alone, as taken from Fig. 5, for $t/H = 0{\cdot}38$. For an air velocity of 100 ft/min the Nusselt number is in every instance less than one-half of that for natural convection alone, but remains dependent on the Grashof number. With an air velocity of 200 ft/min the Nusselt number becomes practically independent of Gr but dependent on the opening size. Such a result is to be expected for high velocities, for then theory requires that the Nusselt number depend only on the Prandtl number and on the Reynolds number VH/ν. It will be noted that

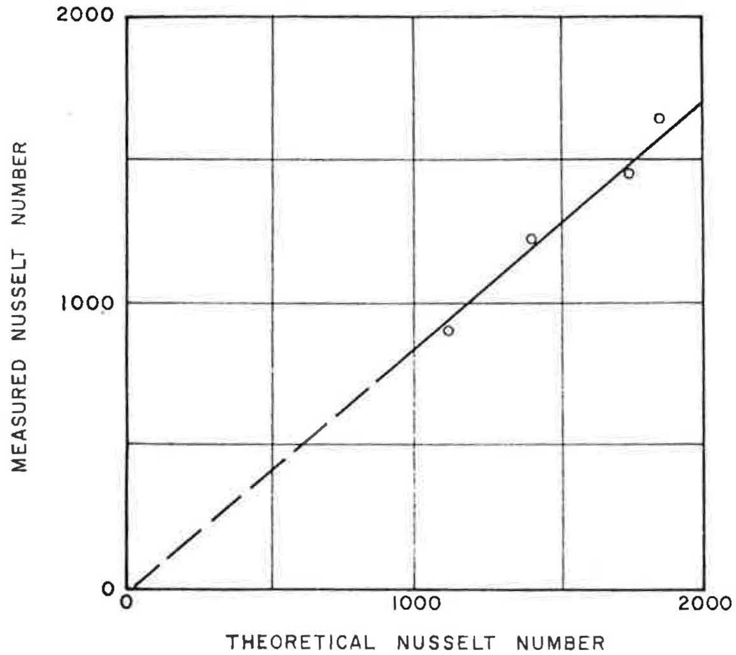

FIG. 6. Natural convection heat transfer across two 3 × 3 in square openings spaced vertically 15 in apart on centers in a 2¼-in thick partition. (The Nusselt numbers are based on a height of 3 in and a total area of 18 sq. in.)

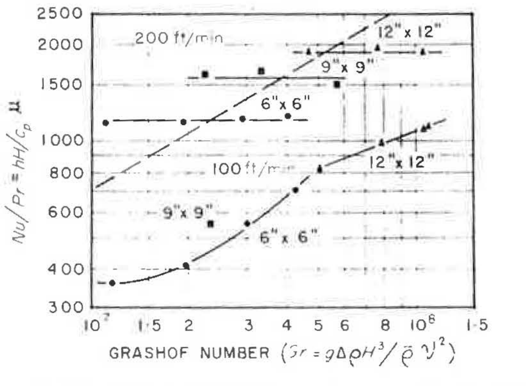

FIG. 7. Effect of a horizontal air flow parallel to the opening and partition. (The dashed curve is that for natural convection with $t/H = 0{\cdot}38$ from Fig. 5.)

the results for $V = 200$ ft/min cross the curve for natural convection alone, i.e. if Gr is large the convection heat transfer with forced convection is less than that of natural convection alone, whereas for low Gr the heat transfer is greater if forced convection is present.

DISCUSSION AND CONCLUSION

The test results for air in natural convection across rectangular openings in vertical partitions are in good agreement with theory. In particular the exponent on the Grashof number in the equation $Nu = BGr^{a}Pr^{b}$ is slightly greater than 0·5 as called for by consideration of the heat-transfer mechanism. Test results for a double opening are also in good agreement with theory. The ratio of wall thickness to opening size also influences the Nusselt number.

It was found that a horizontal forced-air flow parallel to the wall and opening could reduce the convection heat transfer under certain conditions.

The present tests encompass a range of the Grashof number from 10^6 to 10^8 and a range of the ratio of wall thickness to opening height of 0·19–0·75. For air, these results cover most cases of practical interest either for heat or mass transfer. For other fluids having Prandtl or Schmidt numbers greater than about 0·1 the results of the present tests would be expected