DEVELOPMENT OF AN ODE MODEL FEATURING A THREE LEVEL BLEED CONTROL AND AN OFF-LOADING SEQUENCE FOR STANDING COLUMN WELLS

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ABSTRACT

A 2D axisymmetric ODE model that couples heat transfer and groundwater flow in and around a standing column well system was developed by the means of a thermal resistance and capacity network. In this work, the real transient behaviour of the groundwater velocity field and well drawdown is considered by applying the spatial and temporal superposition of the so-called Theis equation, resulting from the time varying bleed flow. The temperature field is obtained by integrating an ODE system. The heat pumps are integrated in the model, thereby allowing the effect of its entering water temperature (EWT) on its capacity and coefficient of performance to be accounted for. A three level bleed control and an off-loading sequence are included in the model. Results demonstrate that the ODE model developed in this paper shows good agreement with the reference solution and that both bleed and off-loading sequence played a key role in maintaining the EWT within a desired operational range. Combined with spatial and temporal superposition techniques, this approach easily allows the evaluation of the thermal and hydraulic interaction of a field of several SCWs.

INTRODUCTION

Ground-coupled heat pump systems (GCHPS) are considered in many countries a reliable and stable solution for providing space heating and air conditioning to commercial and institutional buildings. Among these systems, standing column well (SCW) systems, which use groundwater as heat source/sink, have the potential to deliver much higher heat exchange rate than conventional closed-loop vertical borehole heat exchangers (Yavuzturk and Chiasson, 2002).

In a SCW, groundwater is pumped from the bottom of a deep well, transferred to one or several heat pumps, and then re-injected at the top of the same well (Fig. 1). During peak periods, the performance of the system can be enhanced by discharging part of the pumped water, thereby creating a drawdown that induces groundwater flow to the SCW. This operation, known as bleed, helps maintain the heat pumps EWT within its operational range.

Sometimes, bleeding is not sufficient to maintain the EWT within the limits of the heat pump. In those situations, an off-loading sequence can be initiated and the building's heat pumps are shut down sequentially until the EWT comes back to a suitable temperature.

Owing to the complex underground thermal and hydraulic processes of SCW systems, various numerical models were developed by the means of finite volume and finite element method for its analysis (Deng, 2004; Rees et al., 2004; Abu-Nada et al., 2008; Ng et al., 2011; Croteau, 2011). Deng et al. (2005) also developed a simplified 1D finite difference model to simulate rapidly the thermal response of a SCW. However, in Deng's simplified model, the borehole does not adequately account for the thermal short circuiting between the ascending and descending fluids.

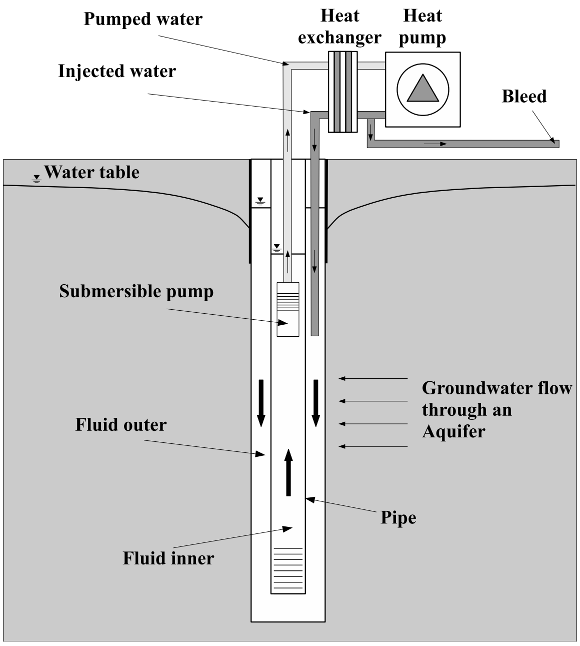

Figure 1: Illustration of a SCW system.

An other way to approach the underground heat transfer problem is to describe the system as a network of thermal resistances and capacities. Numerous authors proposed this approach to simulate the heat flux between the different components of an underground closed-loop heat exchanger (De Carli et al., 2010; Zarrella et al., 2011; Bauer et al., 2011a,b; Pasquier and Marcotte, 2012). Recently, Ramesh and Spitler (2012) used this method in a quasi-two dimensional SCW model to better represent the thermal short circuiting in the borehole and improve Deng's model. In their model, Ramesh and Spitler (2012) implicitly assumed that a drawdown cone around the well is instantly developed in the aquifer and therefore they neglected the storage and/or drainage of the aquifer.

The objective of this paper is to present a 2D axisymmetric thermal resistance and capacity model (TRCM) that couples transient heat transfer and transient groundwater flow in a SCW system. Integration of the transient component of the groundwater flow allow taking into account the temporal evolution of the groundwater velocity field and well drawdown during bleed operations, an original contribution to the field. The model also features a three-level bleed control as well as a heat-pump off-loading sequence. A simplified geometry for this model where the domain extending below the well is neglected (Fig. 2) was chosen to represent the real geometry described in Fig. 1. Note that the modeling of a plate heat exchanger generally installed between the heat pumps and the SCW was not conducted in this paper.

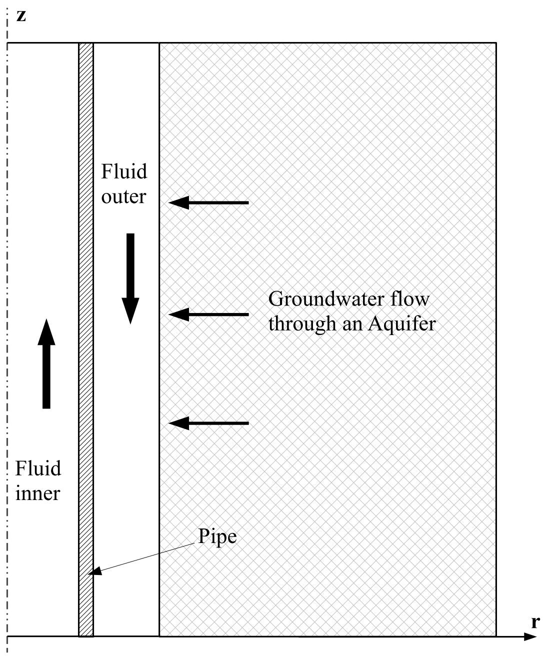

Figure 2: Simplified geometry of a SCW system.

METHODOLOGY

In what follows, the methodology used to construct the TRCM of a SCW system is presented. In order to simplify the complex heat and mass transfer process involved in a SCW, the following assumptions are made:

1. The aquifer is confined, fully saturated, homogeneous, infinite and obeys Darcy's Law.
2. There are no heat source nor vertical groundwater flow in the aquifer.
3. The heat pumps are connected in parallel.
4. When the EWT reaches the temperature limits of the heat pumps, some heat pumps are deactivated. During these conditions, an auxiliary heating or cooling system is used to cover the building energy demand.
5. Only one well is considered, there is no thermal nor hydraulic interaction.
6. The bleeding flow is not reinjected into the aquifer.

Based on the above assumptions, transient groundwater flow and heat transfer models can be established for a single SCW.

Heat transfer model

The temperature T (°C) variation over time t (s) and space for a control volume is based on the rate of internal energy change. Thus, the SCW system is modeled through a series of interconnected thermal resistances and thermal capacities which represents a dynamic system subjected to initial and boundary conditions and can be expressed by:

$$C_j \frac{\mathrm{d}T_j}{\mathrm{d}t} = \sum_{k=1}^{n_j} \frac{T_k - T_j}{R_k} \qquad \forall\, j = 1...n \qquad (1)$$

where n , j, n_j, and k are the total number of nodes in the network, the node index, the number of neighbouring nodes to the node j and the index of the neighbouring node, respectively.

Given the coaxial geometry of this model, the SCW system can be discretized into n_r annular regions. Therefore, the integration of thermal resistances in the radial direction (Rr) allows the different components of the system shown in Fig. 3 to be accounted for. In addition, the model is divided into n_z layers and connected by thermal resistances (Rz) to consider vertical heat transfer. In order to solve Eq. 1, one first needs to evaluate C_j and R_k for each nodes and their spatial distribution.

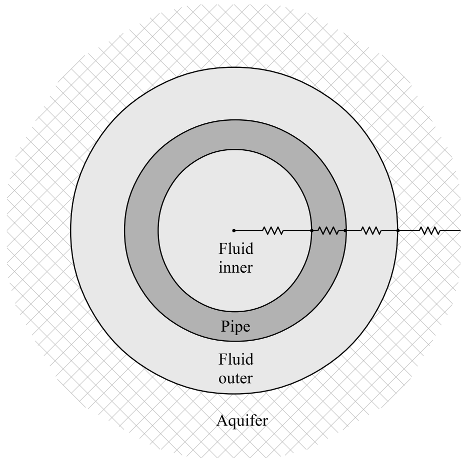

Figure 3: Thermal resistances in the radial direction associated with the different components of the SCW.

The network expressed by Eq. 1 is a system of ordinary differential equation (ODE) which can be easily integrated by commercially available ODE solvers, such as Matlab functions ODE15s and ODE23s (Matlab, 2011), allowing a rapid computation of the temperature at virtually any time and any nodes.

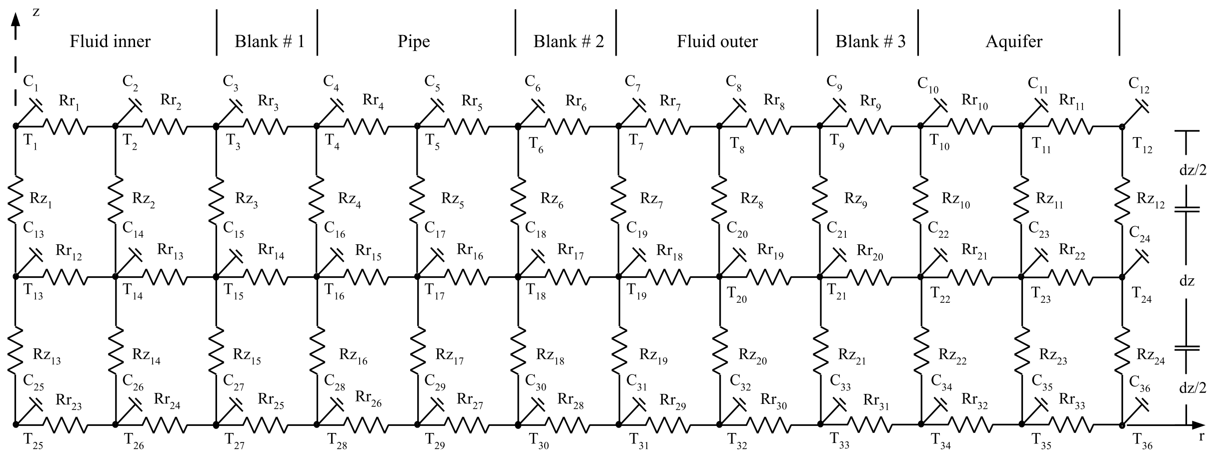

Figure 4: An example of a TRCM with n_{fi} = n_p = n_{fo} = n_a = 2 and n_z = 3 with 3 blank regions. Subscript j refers to the node's index.

Thermal capacity

To simulate transient heat transfer, a thermal capacity is assigned to each node, which corresponds to its surrounding control volume. Therefore, the capacities of each node C_j (J/K) are distributed in the network as following:

$$C_j = \rho_j c_j v_j \qquad \forall\, j = 1...n \tag{2}$$

where ρ_j is the density (kg/m^3) and c_j is the specific heat capacity (J/(kg·K)) associated to the control volume v_j (m^3).

To improve the model accuracy, the diffusive thermal resistances Rr corresponding to each component of the SCW system are equally divided into n sub-regions, thus providing a set of sub-resistances in series. In order to do so, one first need to distribute the nodes. For the inner fluid, the distribution of the nodes is given by:

$$r_{j+1} = r_j + r_{pi}/n_{fi} \qquad \forall\, j = 1...n_{fi} \tag{3}$$

with r_{pi} is the inner radius of the pipe and n_{fi} is the number of regions in the inner fluid (r_1 = 0). Furthermore, the expression for the distance r corresponding to the rest of the nodes (pipe, outer fluid and aquifer) is given by:

$$r_{j+1} = r_j e^{2\pi k \frac{Rr}{n}} \qquad \forall\, j = n_p...n_r \tag{4}$$

where k is the thermal conductivity (W/(m·K)) of the material. Table 1 summarizes the parameters used in Eq. 4 for each component of the SCW.

Table 1: Parameters for each component of the SCW.

SCW components	k (W/(m·K))	Rr (K/W)	n (-)
Inner Fluid	–	Rr_{fi}	n_{fi}
Pipe	k_p	Rr_p	n_p
Outer Fluid	–	Rr_{fo}	n_{fo}
Aquifer	k_a	Rr_{da}	n_a

Vertically, the model is discretized into n_z layers of equal thickness dz. Note that thickness dz for the first and last row in the network is halved.

The nodes obtained by Eq. 3 and 4 are then repeated for every layer. The latter discretization leads to the control area given for node j by:

$$A_j = \frac{\pi(r_j^2 - r_{j-1}^2)}{2} + \frac{\pi(r_{j+1}^2 - r_j^2)}{2} \tag{5}$$

The node's control volume v_j is therefore equal to:

$$v_j = A_j \mathrm{d}z \tag{6}$$

To avoid having capacities from 2 different components mixed within the same control volume, blank regions having a null radial resistance ($Rr = 0$) are squeezed in at interfaces. This ensures that a single material is attributed to a node, which is particularly important when an immobile material is in contact with a moving fluid for instance.

Fig. 4 shows an example of a TRCM with 3 layers for a case where Rr_{fi}, Rr_p, Rr_{fo} and Rr_{da} are divided into 2 sub-resistances in series and where 3 blank regions are introduced ($n_{fi} = n_p = n_{fo} = n_a = 2$ and $n_z = 3$) for a total of 36 nodes, 33 radial thermal resistances and 24 vertical thermal resistances.

Thermal resistance

In a SCW system, heat is transferred through a solid by diffusion and transported by moving groundwater by advection. Due to the different heat transfer mechanisms in the system, advective and diffusive thermal resistance must be assigned accordingly in the network.

For the inner and outer fluid, heat transfer is mostly governed by heat advection. Hence, the required heat flow Q to induce a temperature difference ΔT to a certain amount of fluid in motion is:

$$Q = \dot{V}\rho c \Delta T \tag{7}$$

where $\dot{V}$ is the pumping flow rate (m^3/s), ρ is the fluid's density and c is the specific heat capacity (J/(kg·K)). Thus, Eq. 7 can be rearranged in terms of the inner and outer fluid ν_i (m/s) and ν_o (m/s) to obtain the fluid's advective thermal resistance in the vertical direction Rz:

$$Rz_{fi} = \frac{\Delta T}{Q} = \frac{1}{A_j \nu_i \rho c} \tag{8}$$

$$Rz_{fo} = \frac{\Delta T}{Q} = \frac{1}{A_j \nu_o \rho c} \tag{9}$$

For the groundwater velocity inside the SCW, conservation of mass is used to determine ν_i and ν_o at location z. The fluid's radial thermal resistance varies according to the flow regime. Several correlations can be used to obtain the heat transfer coefficient. To simplify the modeling, it is assumed that the flow inside the SCW is fully turbulent. Thus, the thermal resistance in the radial direction for the inner and outer fluid Rr_{fi} and Rr_{fo} are set at 0^+ (K/W).

For the pipe, only heat diffusion is considered. Thus, the thermal resistance in the radial direction Rr_p is obtained by:

$$Rr_p = \frac{\ln(r_{po}/r_{pi})}{2\pi k_p dz} \tag{10}$$

where r_{po} and r_{pi} are the outer and inner radius of the pipe (m), k_p is the thermal conductivity of the pipe (W/(m·K)) and dz is the thickness of the layer (m).

In the vertical direction, Rz_p associated with heat diffusion through a plane surface A (m^2) is equal to:

$$Rz_p = \frac{dz}{k_p A_j} \tag{11}$$

In the aquifer, both heat diffusion and advection are governing heat transfer. The radial diffusive thermal resistance Rr_{da} in the aquifer is obtained then by:

$$Rr_{da} = \frac{\ln(r_a/r_b)}{2\pi k_a dz} \tag{12}$$

where r_a is the radius of the aquifer, r_b is the radius of the SCW, k_a is there thermal conductivity of the aquifer (W/(m·K)).

Additionally, heat transfer by advection in the aquifer becomes a factor when the system is bleeding part of the pumped water. Therefore, the advective thermal resistance in the aquifer Rr_{aa} is given by:

$$Rr_{aa} = \frac{1}{\nu_D A \rho c} \tag{13}$$

where ν_D is Darcy's velocity of the aquifer in the r direction, the latter being governed by the groundwater model discussed in the next subsection. In order to combine both diffusion and advection, an equivalent thermal resistance Rr_a is obtained under the assumption of parallel resistance, by:

$$\frac{1}{Rr_a} = \frac{1}{Rr_{da}} + \frac{1}{Rr_{aa}} \tag{14}$$

In situations where groundwater is not flowing, Rr_{aa} becomes infinite and $Rr_a = Rr_{da}$. Note that whenever a node contains advective thermal resistances, the downstream portion of the node's network is ignored. Finally, assuming there is no vertical groundwater flow in the aquifer, Rz_a associated with heat diffusion through a plane surface A (m^2) is expressed by:

$$Rz_a = \frac{dz}{k_a A_j} \tag{15}$$

Groundwater flow model

The spatial and time dependant drawdown s (m) created by bleeding can be obtained by combining the spatial and the temporal superposition principle to the so-called Theis analytical equation. This leads to:

$$s(t) = \sum_{i=1}^{n_t} \frac{B(t_i) - B(t_{i-1})}{4\pi K h} \int_{\frac{r^2 S/4\pi K h}{(t-t_i)}}^{\infty} \frac{e^{-u}}{u}\,\mathrm{d}u \tag{16}$$

where B is the bleeding flow rate (m^3/s), h is the SCW's length (m), K is the aquifer's hydraulic conductivity (m/s), S is aquifer's storativity (-), r is the distance from the SCW to the point where the drawdown is observed (m), t is the time (s), t_i is the time corresponding to the beginning (or ending) of a bleeding phase (s) and n_t is the number of superposition at time t. Thus, Darcy's velocity ν_D (m/s) of the aquifer can be determined using Darcy's Law:

$$\nu_D = K\frac{\mathrm{d}s}{\mathrm{d}r} \tag{17}$$

Boundary conditions

The average temperature along the first layer of the inner fluid corresponding to EWT is given by:

$$EWT = \frac{1}{r_{pi}} \int_0^{r_{pi}} T(r)dr \tag{18}$$

Hence, it influences the energy delivered by the heat pump, or its capacity (CAP), and its coefficient of performance (COP). Both CAP (W/unit) and COP (-) affect the temperature variation within the heat pump ΔT, which can be modeled by:

$$\Delta T = (n_{hp} - n_{off}(EWT)) \cdot \frac{CAP(EWT)}{\dot{V}\rho c} \cdot \left(1 \pm \frac{1}{COP(EWT)}\right) \tag{19}$$

where $\dot{V}$ is total the flow rate (m^3/s), n_{hp} and n_{off} are the total number of heat pumps and the number of offloaded heat pumps, respectively. In the model presented in this work, the heat pumps were modeled using commercially available performance chart, which

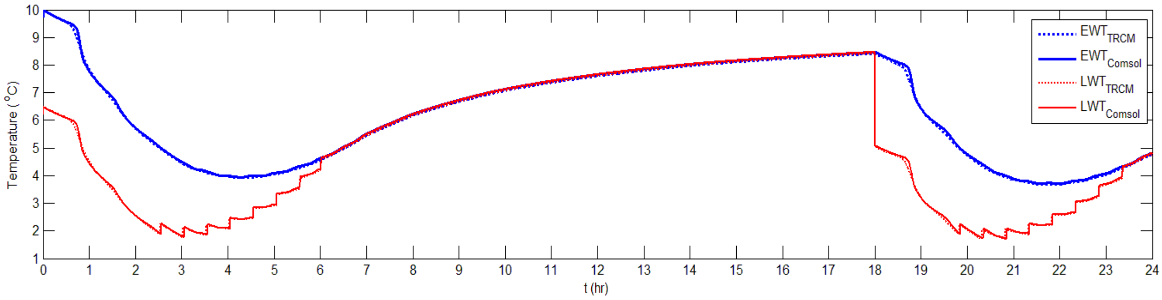

Figure 5: Comparisons between the TRCM and the FEM for EWT and LWT.

allows the evaluation of the performance as well as the energy delivered by a heat pump at any time step.

The node's temperature along the first layer of the outer fluid is set equal to the leaving water temperature (LWT) of the heat pumps, given by:

$$LWT = T_j = EWT \pm \Delta T$$
$$\forall j\ [n_{fi} + n_p + 1, n_{fi} + n_p + n_{fo} + 1] \qquad (20)$$

Furthermore, the average temperature along the last layer of the inner and outer fluid are assigned equal. This setup ensures the recirculation of groundwater inside the SCW. Finally, the temperature at the far field is set constant.

Computer algorithm

A function managing the three-level bleed control and the off-loading sequence is being called at each time step taken by the ODE solver. The function compares the EWT with a set of 4 temperature thresholds assigned by the user (3 for the bleed control and 1 for the offloading sequence).

The first three temperature thresholds are used to modulate B, the bleeding rate, and reduce the power consumption of submersible pumps. Whenever the EWT crosses one of the first three limits, the corresponding solver's time steps is assigned to t_i and Rr_{aa} is calculated based on Eq. 13.

If, despite the activation of the third bleed level, the EWT continues to decrease and reaches the last threshold, an offloading sequence is initiated, which deactivates sequentially the heat pumps, therefore protecting them against possible mechanical failures. At that moment, the current solver time step is assigned to the t_{off} variable. At that moment, an off-loading sequence is initiated and a first heat pump is deactivated ($n_{off} = 1$).

If the EWT remains below that temperature 30 minutes after the time corresponding to t_{off}, n_{off} is raised by 1 and the current time t is assigned as the new t_{off} value. The process is repeated until the number of offloaded heat pumps reaches the total number of installed heat pumps (n_{hp}). If the EWT rises back above that temperature during this process, n_{off} is set back to 0, allowing the re-opening of all the heat pumps.

Table 2: Dimensions of the SCW system.

Parameters	Value (m)
r_{pi}	0.08
r_{po}	0.09
r_b	0.15
r_a	5
h	250

Main computation steps

To summarize the methodology, the main computation steps are listed below:

1. Define the SCW and system dimensions r_{pi}, r_{po}, r_b, r_a and h.
2. Initialise number of discretization n_{fi}, n_p, n_{fo}, n_a and n_z.
3. Use Eq. 2 to derive the thermal capacities C_j for each node in the system.
4. Use Eqs. [8 to 14] to calculate the thermal resistances R_k.
5. Solve Eq. 1 under the constraints presented in the previous subsections.

SIMULATION

In order to demonstrate the model proposed in this paper, a simulation has been conducted with the parameters presented in Tables 2 and 3. The simulation considers 6 hours of constant heating followed by 12 hours of no-load and another 6 hours of constant heating. The building load of 120 kW is provided by a hybrid system which consist of 8 heat pumps (nominal CAP of 14 kW (4 TONs) per heat pump) and an auxiliary heating system, the latter having a COP of 1. The initial ground temperature T_0 is set at 10 °C.

Table 3: Thermal and hydraulic properties.

	Inner Fluid	Pipe	Outer Fluid	Aquifer	Units
ρ	1000	950	1000	2500	kg/m^3
c_p	4200	2500	4200	800	$J/(kg \cdot K)$
k	-	0.5	-	2.2	$W/(m \cdot K)$
K	-	-	-	1e-6	m/s
S	-	-	-	0.24	-

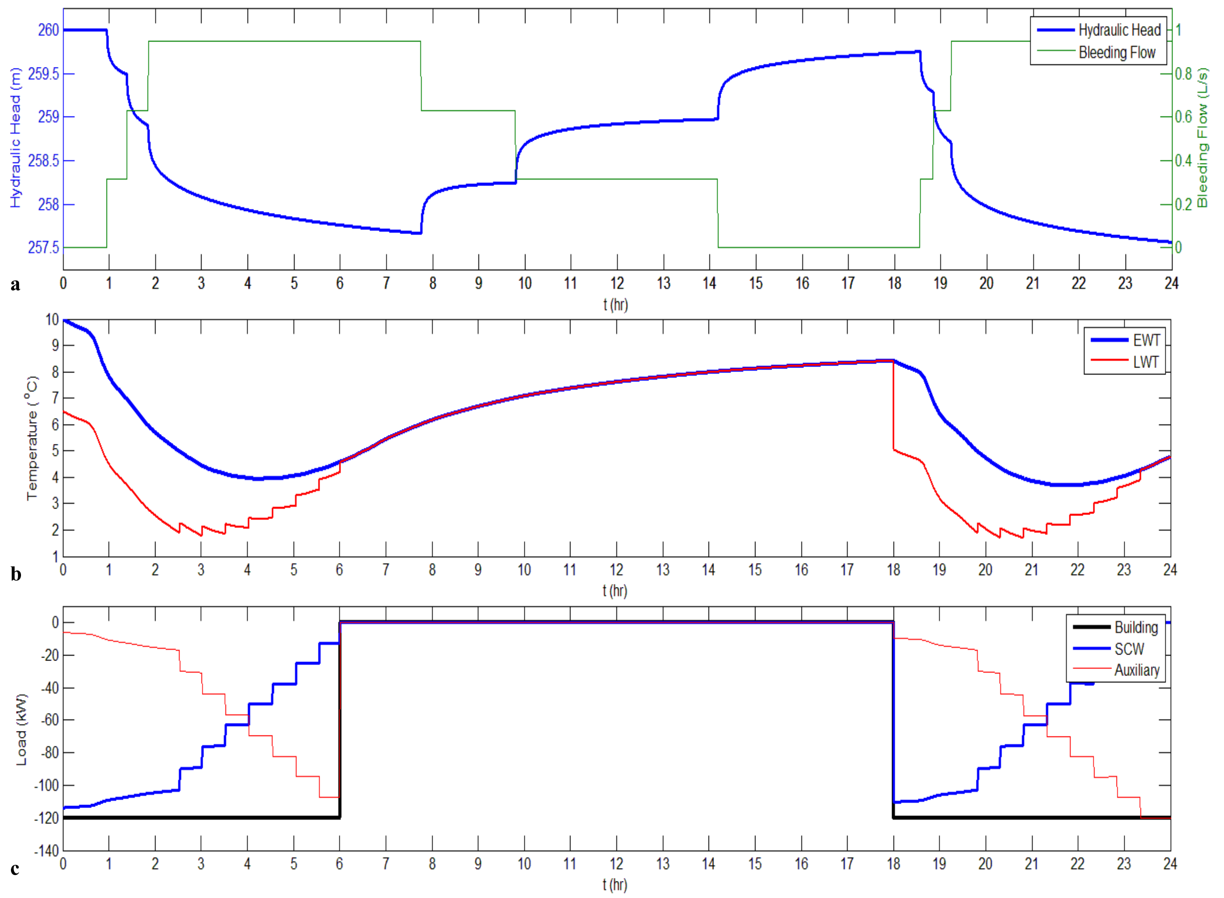

Figure 6: Temporal evolution of a) the hydraulic head and bleeding flow, b) entering and leaving water temperature, and c) use of the heat pumps and auxiliary heating system.

Table 4: Description of the scenarios.

Scenario #	Bleed	Off-Loading Seq.
1	Yes	Yes
2	No	Yes
3	Yes	No
4	No	No

The total pumping rate $\dot{V}$ is set at 6.3e-3 m^3/s. Hence, the three-level bleed control is set at 5, 10 and 15 % of the total pumping rate and initiated at 8, 7, and 6 °C, respectively. The offloading sequence starts at 5 °C. To improve the model accuracy, the SCW system is discretized into 65 annular regions (n_r = 65) and 31 layers (n_z = 31).

In order to validate the approach proposed in this paper as well as the methodology used to implement the three-level bleed control and the offloading sequence, the simulation mentioned above has also been conducted with the finite element model developed by Nguyen et al. (2012) within the COMSOL Multiphysics 4.3a environment and both the EWT and LWT are compared. Furthermore, 4 scenarios (Table 4) have been conducted in order to evaluate the combined and individual effectiveness of bleed and heat-pump offloading.

RESULTS AND DISCUSSION

Comparisons of EWT and LWT from the TRCM and the finite element model for time steps of 30 seconds are shown in Fig. 5. Results indicate very good agreement between the two models. The average difference for EWT and LWT is about 0.05 °C which is an acceptably small difference. Overall, the model proposed in this work accurately reproduces the results of the reference finite element model.

Fig. 6 a. shows the hydraulic head inside the SCW and the corresponding bleed flow rate over time. Both hydraulic head and bleed flow rate variations corresponding to EWT at 8, 7 and 6 °C (Fig. 6 b.) shows that the implementation of the bleed operation was successfully achieved in this model. It also indicates that the off-loading sequence accurately started at 5 °C and that a heat pump subsequently stopped every 30 minutes. One can also note that once the EWT went above the 5 °C limit, all the heat pumps were allowed to reopen if heating was required.

Fig. 6 c. shows the thermal loads provided by the heat pumps and the auxiliary heating system. The combined heating power of both subsystems correctly corresponds to the overall building heating load at each time step.

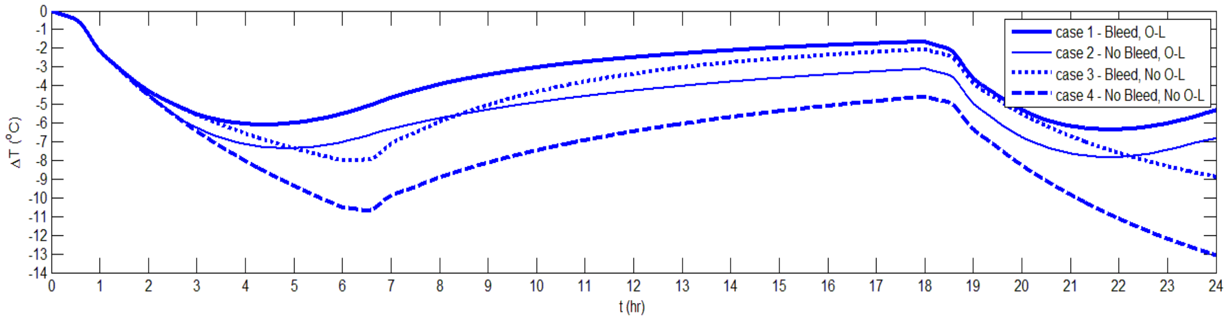

Figure 7: Illustration of the ΔT corresponding to the EWT drop for the different scenarios presented in Table 4. The lowest ΔT corresponds to the best scenarios.

Fig. 7 shows the ΔT corresponding to the EWT drop for the different scenarios presented in Table 4. As expected, the combination of both three-level bleed control and off-loading sequence (Scenario 1) minimizes the EWT drop among all 4 scenarios. In Scenario 2, where only the offloading sequence is considered, the EWT drops quickly at the beginning of the simulation, then stabilises when the offloading sequence initiates. During the no-load period however, the EWT does not recover as fast. Furthermore, if only the three-level bleed control is considered (Scenario 3), the EWT drops more as a result of the constant heat pump operation during the heating period. However, the EWT recovers sharply during the no-load period. Thus, this demonstrates the effectiveness of the three-level bleed control. Finally in Scenario 4, neither of the three-level bleed control and the offloading sequence are considered and the EWT drops by 13 °C after 24 hours. In this case, the SCW system is clearly not sustainable.

CONCLUSION

In this paper, a 2D axisymmetric TRCM of a SCW that integrates a three level bleed control and an off-loading sequence was developed by the means of a ODE system. This model has been compared against a finite element model and shows very good agreements.

The results show that this numerical model can successfully evaluate the EWT over time, therefore allowing an evaluation of the load provided by the heat pumps and their energy consumption. Four scenarios were conducted to evaluate the effectiveness of both bleed control and offloading sequence. Results show that both bleed and off-loading sequence played a key role in maintaining the EWT within a desired operational range.

The current model can be used to forecast, for a homogeneous aquifer, the performance of the planned design and control system under different heat load scenarios. When used in conjunction with spatial superposition principle, the model proposed in this work allows easily the calculation of the thermal and hydraulic response of a field of several SCWs.

NOMENCLATURE

A	area (m^2)
B	bleed flow (m^3/s)
C	thermal capacity (J/K)
c	specific heat capacity (J/(kg·K))
COP	coefficient of performance (-)
CAP	capacity (W/unit)
K	hydraulic conductivity (m/s)
k	thermal conductivity (W/(m·K))
H	hydraulic head (m)
h	lenght of SCW (m)
n_{hp}	number of heat pump (-)
n_j	sum of neighbouring nodes to the node j (-)
n_{off}	number of offloaded heat pump (-)
n_r	total number regions (-)
n_t	number of superposition at time t (-)
n_z	total number layers (-)
Q	heat flow (W)
r	radius (m)
Rr	radial thermal resistance (K/W)
Rz	vertical thermal resistance (K/W)
s	drawdown (m)
S	storativity (-)
t	time (s)
tb	time of the beginning (or ending) of a bleeding operation (s)
t_{off}	start time of the offloading (s)
T	temperature (°C)
z	depth (m)
$\dot{V}$	flow rate (m^3/s)
ρ	density (kg/m^3)
ν	velocity (m/s)

Subscripts

fi	inner fluid
p	pipe
fo	outer fluid
a	aquifer
aa	advection in aquifer
da	diffusion in aquifer
D	Darcy
i	time step index
j	node index
k	neighbouring node index

Acronyms

EWT	entering water temperature
LWT	leaving water temperature
SCW	standing column well
TRCM	thermal resistance and capacity model

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